Uncountability of $\mathbb{R}$

We will prove that $(0,1)$ is uncountable. To do this, we write x in the base 2. Note that we may have two expressions for rational numbers. To fix the representation, for such numbers, we choose the expression ending with 0's, e.g, $1/2 = .10000\cdots$, not $.01111\cdots$. Assume that $(0,1)$ is countable. Then we can write $(0,1) = \{r_1, r_2, r_3, \cdots\}$. It suffices to choose r_1 with the property that the first digit after the decimal point is 1 and r_2 having the property that the second digit after the decimal point is 0. That is

$$r_1 = .1 * * * \cdots, \quad r_2 = . * 0 * * * \cdots.$$

In view of the process we discussed in the class, we can construct

$$r = .01 * * * \cdots$$

which differs from any number of $\{r_1, r_2, r_3, \cdots\}$. It is clear that $r \in (0,1)$. I choose r_1 and r_2 in this way to avoid that r ends up at end points 0 or 1.