Homework #6

1. Show that the Cantor set is a uncountable measure zero set.

2. Let $\{R_j\}_{j=1}^N$ be non-overlapping rectangles, then $| \cup_{k=1}^N R_j| = \sum_{j=1}^N |R_j|$.

3. Let A and B two disjoint compact sets of $\mathbb{R}^n$. Show that $\text{dist}(A, B) > 0$.

4. Prove that every open set in $\mathbb{R}^n$ can be written as a countable union of non-overlapping (closed) cubes.

5. Prove that outer measure is *translation invariant*, i.e., if $E_h = \{x + h; x \in E\}$ is the translate of E by $h \in \mathbb{R}^n$, then $|E + h|_e = |E|_e$.

6. Let Z be a subset of $\mathbb{R}^1$ with measure zero. Show that the set $\{x^2 : x \in Z\}$ also has measure zero.