I answer some of your questions in this note.

1. P. 560, 101(a). We first observe that

$$x_{n+1} - x_n = \frac{x_n}{2} + \frac{1}{x_n} - x_n = \frac{1}{x_n}(1 - \frac{x_n^2}{2}).$$

Now we look at the recursive formula

$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}.$$

So we consider the function

$$f(x) = \frac{x}{2} + \frac{1}{x}.$$

Solving $f'(x) = 0$, we found that the critical points are $\pm\sqrt{2}$. Since $f''(x) = \frac{2}{x^3}$, $f(\sqrt{2}) = \sqrt{2}$ is the global minimum of $f(x)$ in $x > 0$. In other words, all $x_n \geq \sqrt{2}$ for $n \geq 1$. Consequently,

$$x_{n+1} - x_n \leq 0 \quad \text{for} \quad n \geq 1.$$

So $\{x_n\}$ is a nonincreasing sequence and $x_n \geq \sqrt{2}$ for all $n \geq 1$. A noninreasing sequence which is bounded below converges. To find $\lim_{n\to\infty} x_n$, we set

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} (\frac{x_n}{2} + \frac{1}{x_n}) = \frac{x}{2} + \frac{1}{x}.$$

Thus, $x = \sqrt{2}$.

2. To find the orthogonal family of the curves $y^2 = cx^3$. We could do it in the following way: write $y^2/x^3 = c$ and differentiate both sides to get

$$\frac{2y'y}{x^3} - \frac{3y^2}{x^4} = 0.$$

So

$$y' = \frac{3y}{2x}.$$

This is the slope of the tangent $y^2 = cx^3$. However, if we do

$$2yy' = 3cx^2 \Rightarrow y' = \frac{3cx^2}{2y}.$$

This is not the right formula for the tangent of $y^2 = cx^3$. Here c is not a constant, it depends on y and x.