Homework #1: Consider $t \in [x_0, x]$. Let

$$F(t) = \frac{(x-t)^n}{n!} \quad \text{and} \quad G(t) = f(x) - \sum_{k=0}^{n-1} \frac{f^{(k)}(t)}{k!}(x-t)^k.$$

Using the generalized mean value theorem (Cauchy mean value theorem) to show the Taylor formula:

$$f(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + \frac{f^{(n)}(c)}{n!}(x-x_0)^n$$

for some $c \in (x_0, x)$. You need to compute $F'(t)$ and $G'(t)$. To compute $G'(t)$, it suffices to compute

$$\frac{d}{dt}\left(\frac{f^{(k)}(t)}{k!}(x-t)^k\right) = \frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(k)}(t)}{(k-1)!}(x-t)^{k-1}, \quad 1 \le k \le n-1.$$

For $k=0$, we simply have $f'(t)$. Sum them up and see what you got (a lots of terms will be canceled out).