

$$\begin{aligned}\int \sec x dx &= \int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx = \int \frac{\cos x}{1 - \sin^2 x} dx = \int \frac{1}{1 - u^2} du (u = \sin x) \\ &= \int \frac{1}{2} \left(\frac{1}{1 - u} + \frac{1}{1 + u} \right) du = \frac{1}{2} (\ln |1 + u| - \ln |1 - u|) + C \\ &= \frac{1}{2} (\ln |1 + \sin x| - \ln |1 - \sin x|) + C.\end{aligned}$$