$$\int \sec x dx = \int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx = \int \frac{\cos x}{1 - \sin^2 x} dx = \int \frac{1}{1 - u^2} du (u = \sin x)$$

$$= \int \frac{1}{2} (\frac{1}{1-u} + \frac{1}{1+u}) du = \frac{1}{2} (\ln |1+u| - \ln |1-u|) + C$$

$$= \frac{1}{2} (\ln |1 + \sin x| - \ln |1 - \sin x|) + C.$$