As we have mentioned in the class, $\mathbf{R}$ is an ordered field. Consider the subset of $\mathbf{R}$

$$\mathbf{R}_+ := \{x \in \mathbf{R} : x \geq 0\}.$$

It is clear that $\mathbf{R}_+$ is an inductive set. By the definition of $\mathbf{N}$, we must have $\mathbf{N} \subset \mathbf{R}_+$. In other words, $0 < n$ for all $n \in \mathbf{N}$ with $n \neq 0$. Therefore, 0 is the smallest element of $\mathbf{N}$.