pf of Thm 11.9 :

Choose $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$ such that $a < a_n < b_n < b$ and $a_n \downarrow a$, $b_n \nearrow b$ as $n \to \infty$.

For each $n \in \mathbb{N}$, define $F_n : [c, d] \to \mathbb{R}$ by

$$F_n(y) = \int_{a_n}^{b_n} f(x, y)\,dx \quad \text{for } y \in [c, d].$$

Then by Thm 11.5, ($\because f_y$ is conti. on $[a_n, b_n] \times [c, d]$)

$$F_n'(y) = \int_{a_n}^{b_n} f_y(x, y)\,dx \quad \text{for } y \in [c, d].$$

Obviously,

$$F_n \to F \quad \text{as } n \to \infty.$$

& by the uniform convergence of ϕ on $[c, d]$,

$$F_n' \to \phi \quad \text{as } n \to \infty$$ ($\because \phi(y)$ converges uniformly [illegible] $d]$)

uniformly on $[c, d]$.

Hence, by Thm 7.12,

$$\phi(y) = F'(y).$$

proved by Mr. 呂.