pf of Thm 11.9:

Choose \( \{a_n\}_{n=1}^\infty, \{b_n\}_{n=1}^\infty \) such that \( a < a_n < b_n < b \) and
\[ a_n \searrow a, \quad b_n \nearrow b \quad \text{as} \quad n \to \infty. \]

For each \( n \in \mathbb{N} \), define \( F_n : [c, d] \to \mathbb{R} \) by
\[ F_n(y) = \int_{a_n}^{b_n} f(x, y) \, dx \quad \text{for} \quad y \in [c, d]. \]

Then, by Thm 11.5, \( f_y \) is continuous on \([a_n, b_n] \times [c, d]\)
\[ F'_n(y) = \int_{a_n}^{b_n} f_y(x, y) \, dx \quad \text{for} \quad y \in [c, d]. \]

Obviously,
\[ F_n \to F \quad \text{as} \quad n \to \infty. \]

By the uniform convergence of \( f \) on \([c, d]\),
\[ F'_n \to \phi' \quad \text{as} \quad n \to \infty \]
uniformly on \([c, d]\).

Hence, by Thm 7.12,
\[ \phi(y) = F'(y). \] proved by Mr. B.