

Solutions to Advanced Calculus Midterm 1

5. Since $g : [c, d] \rightarrow \mathbb{R}$ is one-to-one and continuous, $g^{-1} : g([c, d]) \rightarrow [c, d]$ exists and is continuous as well. On the other hand, both g and g^{-1} are strictly monotone. Therefore, $g([c, d])$ is a bounded closed interval in \mathbb{R} . In other words, g^{-1} is uniformly continuous on $g([c, d])$. Note that $f(x) = (g^{-1} \circ h)(x)$ and h is uniformly continuous on (a, b) . Thus, we can choose a $\delta > 0$ small enough such that if

$$|x - y| < \delta \Rightarrow |h(x) - h(y)| < \tilde{\delta} \Rightarrow |(g^{-1} \circ h)(x) - (g^{-1} \circ h)(y)| < \varepsilon$$

for all $x, y \in (a, b)$.

6. Step (2) is incorrect. According to the definition, if $x \in \bigcup_{j \in \mathbb{N}} E_j$ then $x \in E_j$ for some $j \in \mathbb{N}$. However, elements of $\mathbb{N} \times \mathbb{N} \times \cdots$ do not belong to any E_j .