Differential Geometry Homework #6 due 11/23

1. Find and fix the (minor) mistake in the statement of Exercise # 2 in Boothby p. 59. Then prove the correct statement.

2. Prove that $\mathbf{RP}^n$ is a smooth manifold by giving a differentiable structure on it.

3. Let $\mathfrak{M}_{mn}(\mathbb{R})$ be the space of all real $m \times n$ matrices and $\mathfrak{M}^k_{mn}(\mathbb{R})$ be the subset of all those $m \times n$ matrices whose rank is $\geq k$. Show that $\mathfrak{M}^k_{mn}(\mathbb{R})$ is a differentiable manifold. (Boothby # 3, p. 59)

4. Let $\mathbb{S}^n$ be the n-sphere with its standard differentiable structure (as given in Prob. **1**). Define $F : \mathbb{S}^n \mapsto \mathbb{S}^n$ the antipodal map $F(x) = -x$. Show that F is smooth by computing the coordinate representation $\hat{F}$ of F in its standard differentiable structure.

5. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be the atlases for $\mathbb{R}$ defined by $\mathcal{A}_1 = \{(\mathbb{R}, id)\}$ and $\mathcal{A}_2 = \{(\mathbb{R}, t^3)\}$. Let $f : \mathbb{R} \mapsto \mathbb{R}$ be any function. Determine necessary and sufficient conditions on f such that it will be:
(a) a smooth map $(\mathbb{R}, \mathcal{A}_2) \mapsto (\mathbb{R}, \mathcal{A}_1)$;
(b) a smooth map $(\mathbb{R}, \mathcal{A}_1) \mapsto (\mathbb{R}, \mathcal{A}_2)$.