1. Let $U$ be an open set of $\mathbb{R}^n$ and $p \in U$. Show that the tangent space $T_p(U)$ of $U$ at $p$ is a vector space of dimension $n$.

2. Let $p \in S^n \subset \mathbb{R}^{n+1}$ and define
   \[
   T_p(S^n) = \{ \langle s \rangle_p \in T_p(\mathbb{R}^{n+1}) | s : (-\epsilon, \epsilon) \to \mathbb{R}^{n+1} \text{ has image}(s) \subset S^n \}.
   \]
   Prove that $T_p(S^n)$ is the linear subspace of $\mathbb{R}^{n+1} = T_p(\mathbb{R}^{n+1})$ consisting of all $v \perp p$.

3. Let $G_p^k$ be the germs of $C^k(U, p)$ functions. Denote $G_p^\infty = G_p$. Is the identification $T(G_p^k) \cong T_p(U)$ still valid when $k < \infty$?

4. Let $G_p^* \subset G_p$ be the kernel of the evaluation map $e_p$ and let $G_p^{**} \subset G_p^*$ be the vector subspace spanned by the germs of functions $gf$, where $g, f \in C^\infty(U, p)$ and $f(p) = g(p) = 0$. Prove that the quotient space $G_p^*/G_p^{**}$ is canonically isomorphic to the vector space dual of $T_p(U)$. The quotient space $G_p^*/G_p^{**}$ is defined in the sense that for $f_1, f_2 \in G_p^*$, $f_1 \sim f_2$ whenever $f_1 - f_2 \in G_p^{**}$.