Differential Geometry Homework #2 due 10/12

1. Let $X = [0, 1] \times [0, 1]$. Define the equivalence relation $\sim$ on X by $(x, 0) \sim (x, 1)$, $(0, y) \sim (1, y)$, and $(x, y) = (x, y)$ for $0 < x < 1$, $0 < y < 1$. Show that $X/\sim$ is homeomorphic to $T^2 = \mathbb{S}^1 \times \mathbb{S}^1$.

2. Define $f : \mathbb{S} \mapsto \mathbb{R}^4$ by the formula

$$f(x, y, z) = (yz, xz, xy, x^2 + 2y^2 + 3z^2).$$

Prove that f passes to a well-defined, topological imbedding

$$\bar{f} : \mathbb{R}P^2 \hookrightarrow \mathbb{R}^4.$$

(Conlon, 1.5, Exercise (1))

3. Show that topologist's sine curve is an immersed, but not imbedded, manifold in $\mathbb{R}^2$.

4. (1) Let X be compact and $X/\sim$ be a quotient space obtained from X. Show that $X/\sim$ is compact, too. (2) Let $f : X \mapsto Y$ be continuous, one-to-one, and onto, where X is compact and Y is Hausdorff. Show that f is a homeomorphism.