Differential Geometry Homework #10 due 12/21

1. Let Γ be a discrete group acting smoothly on the manifold M. Show that Γ acts properly discontinuously on M if and only if any two points x, x' in M have neighborhoods U and U' such that the set $\{h \in \Gamma : hU \cap U' = \emptyset\}$ is finite.

2. Let G be a Lie group acting smoothly on a manifold M. Prove that each orbit is an immersed submanifold of M.

3. (Boothby, p.103 #2) Show that $\pi_2 : \tilde{M}/\tilde{\Gamma} \to M$ as defined in class is a covering map.

4. (Boothby, p.103 #3) Show that the covering transforms form a group and that if $x, y \in \tilde{M}$, a covering manifold of M, then there is at most one covering transform taking x to y. Show further that if $\tilde{\Gamma}$ is transitive on $\pi^{-1}(p)$ for some $p \in M$, then it is transitive for every p.

5. (Boothby, p.103 #5) Let $\pi : \tilde{M} \to M$ be a covering and $F : [a, b] \to M$ a continuous curve from $F(a) = p$ to $F(b) = q$. If $x_0 \in \pi^{-1}(p)$, show that there is a unique continuous curve $\tilde{F} : [a, b] \to \tilde{M}$ such that $\tilde{F}(a) = x_0$ and $\pi \circ \tilde{F} = F$.

6. Let $\pi : \tilde{M} \to M$ be a covering map. Show that π is open.