

Advanced Algebra II

Homework 9

due on May. 25, 2007

- (1) Given a non-singular cubic $X \subset \mathbb{P}^2$. Show that after linear change of coordinates, X can be written as $\mathcal{V}(y^3 - x(x - z)(x - \lambda z))$ for some $\lambda \neq 1, 0$.
- (2) Given a curve $X = \mathcal{V}(x^5 + y^5 + z^5) \subset \mathbb{P}^2$. Compute its canonical divisor.
- (3) Given a complex torus $E_\tau := \mathbb{C}/\Lambda_\tau$, where Λ_τ denote the lattice $\mathbb{Z} + \mathbb{Z}\tau$. It's well-known that the endomorphism, i.e $\phi : E_\tau \rightarrow E_\tau$, are induced by $f_c : \mathbb{C} \rightarrow \mathbb{C}$ that $f_c(z) \mapsto cz$ for some $c \in \mathbb{C}$. Determine all f_c that $f_c(\Lambda_\tau) \subset \Lambda_\tau$ for all τ .
- (4) Let $\mathcal{H} := \{z \in \mathcal{B} | \text{Im}(z) > 0\}$ be the upper half plane of \mathbb{C} . Consider the following action $GL_2(\mathbb{R}) \times \mathcal{H} \rightarrow \mathcal{H}$ by $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \tau \mapsto \frac{a\tau + b}{c\tau + d}$. Describe all stabilizers.