Advanced Algebra II

Homework 8

due on May. 18, 2007

In this exercise, k always denote an algebraically closed field.

(1) * Complete the exercises and incomplete proofs in the note.
(2) * Let $(R, \mathfrak{m})$ be a Noetherian local domain. Show that $\dim R \leq \dim_k \mathfrak{m}/\mathfrak{m}^2$.
(3) Prove the associative law for the group structure on non-singular cubics.
(4) Determine the singularities and its embedded dimension of $\mathcal{V}(x^2 + y^3 + z^5) \subset \mathbb{A}^3$.
(5) Determine the singularities and its embedded dimension of $\mathcal{V}(xz - y^2) \subset \mathbb{A}^3$.
(6) Prove Pascal's theorem.
(7) Let F be a homogeneous polynomial in $k[x, y, z]$ of degree d. Show that $dF = xF_x + yF_y + zF_z$.
(8) Consider $F = \mathcal{V}(y^2z - x^3 + xz^2)$ and $G = \mathcal{V}(xy - z^2)$ in $\mathbb{P}^2$. Determine the intersection multiplicities at each intersection point and verify Bezout's theorem.