

# Advanced Algebra II

## Homework 7

due on May. 11, 2007

- (1) Consider the map  $\varphi : \mathbb{P}^1 \rightarrow \mathbb{P}^2$  by  $\varphi(s : t) = (s^2 : st : t^2)$ . Show that its image is  $\mathcal{V}(xz - y^2) \subset \mathbb{P}^2$ . Moreover show that  $\varphi$  gives an isomorphism to its image.
- (2) Show that there is morphism from  $\mathbb{P}^2 \rightarrow \mathbb{P}^1$ . Also there is no morphism from  $\mathbb{P}^n \rightarrow \mathbb{P}^m$  for  $n > m$ .
- (3) Every varieties can be covered by open subsets which are affine varieties.
- (4) Let  $X := \mathbb{A}^2 - \{0\}$ . Determine  $\mathcal{O}(X)$ . And show that  $X$  is not affine.
- (5) Given varieties  $X, Y$  such that there are open sets  $U \subset X, V \subset Y$  and  $U \cong V$ . Then  $K(X) \cong K(Y)$ . Is the converse true? For example,  $X = \mathbb{A}^n, Y = \mathbb{P}^n$ .