Advanced Algebra II

Homework 6

due on Apr. May. 4, 2007

In this exercise, k always denote an algebraically closed field.

(1) * Complete the exercises and incomplete proofs in the note.
(2) Let R be a finitely generated k-algebra, which is a domain. Let $\mathfrak{p} \lhd R$ be a prime ideal. Show that $\dim R = \dim R/\mathfrak{p} + ht(\mathfrak{p})$.
(3) Consider the map $\varphi : \mathbb{A}^2 \to \mathbb{A}^{d+1}$ by $\varphi(s,t) = (s^d, s^{d-1}t, ..., t^d)$. Show that $im(\varphi)$ is an affine variety. Determine its coordinate ring and its dimension.
(4) Let $f, g \in k[x,y]$ be polynomials of degree $m, n \geq 1$ respectively with no common factors. Show that there are at most mn common zero.
(5) As a set, $\mathbb{A}^2$ can be identify with $\mathbb{A}^1 \times \mathbb{A}^1$. However, the Zariski topology on $\mathbb{A}^2$ is different from the product topology of Zariski topology on $\mathbb{A}^1$.
(6) Given a ring homomorphism $f : R \to S$. Show that there is an induced $f^{\#} : \mathrm{Spec}(S) \to \mathrm{Spec}(R)$, which is continuous.
(7) *Determine the set $\mathrm{Spec}(\mathbb{Z}[x]$.
Consider $f : \mathbb{Z} \to \mathbb{Z}[x]$ and $g_p : \mathbb{Z}[x] \to \mathbb{Z}_p[x]$. Describe the fibers of the induced maps $f^{\#}$, $g_p^{\#}$.