Advanced Algebra II

Homework 3

due on Mar. 23, 2007

(1) Determine $\mathbb{Z}_m \otimes_{\mathbb{Z}} \mathbb{Z}_n$.

(2) Let $M = M_1 \oplus M_2$. Prove that M is flat if and only if both M_i are flat.
What can you say if $M = \oplus_{i \in I} M_i$ with general index set I.

(3) Let R be a local ring and M is a finitely generated flat R-module. Then M is free.
In fact, if $\{x_1, ..., x_n\} \subset M$ such that they form a basis in $M/\mathfrak{m}M$ over $R/\mathfrak{m}$, then it forms a basis of M.

(4) Let $0 \to M_1 \to M_2 \to M_3 \to 0$ be an exact sequence of R-modules. If both M_1, M_3 are finitely generated then so is M_2.

(5) * Tensor product commutes with direct limit. That is, $\varinjlim (M_i \otimes N) \cong (\varinjlim M_i) \otimes N$.

(6) A ring S is said to be an R-algebra if there is a ring homomorphism $R \to S$. Show that S is an R-module.

(7) Let S be a flat R-algebra and M be a flat S-module. Then M is a flat R-module.

(8) * Complete the exercises and incomplete proofs in the note.