Advanced Algebra II

Homework 11

due on Jun. 8, 2007

(1) Let $\mathfrak{D}$ be a Dedekind domain with quotient field K. For a ideal $\mathfrak{a} \lhd \mathfrak{D}$, show that $\mathfrak{a}^{-1}\mathfrak{a} = \mathfrak{D}$, where $\mathfrak{a}^{-1} := \{x \in K | x\mathfrak{a} \subset \mathfrak{D}\}$.

(2) Let $L/\mathbb{Q}$ be a number field. Show that the discriminant d_L is independent of choice of basis. And also $\operatorname{sgn}(d_L) = (-1)^s$. Where $2s$ is the number of complex embedding $L \to \mathbb{C}$.

(3) Consider the quadratic field extension $\mathbb{Q}(\omega)/\mathbb{Q}$. And let $\mathcal{O}$ be the ring of algebraic integers in $\mathbb{Q}(\omega)$. Determine all ideals in $\mathcal{O}$ with absolute norm ≤ 4. And show that all such ideals are principal.

(4) Keep the notation as in Problem # 3. Is $\mathcal{O}$ an Euclidean domain?