Advanced Algebra II

Final Examination

due on Jun. 25, 2007 (Monday)

I. Do following 2 problems.

(1) Write the definition of following terms and explain in a few words.
   1. Noetherian modules.
   2. Semisimple Lie algebra.

(2) State and prove the following theorems.
   1. Weierstrass preparation theorem.
   2. Hilbert's Nullstellensatz.

II. Do any 3 of following 4 problems.

(1) Let $\varphi : R \to S$ be a ring homomorphism. Let $\mathfrak{p} \lhd R$ be a prime ideal. Compare $\dim(S \otimes (R_{\mathfrak{p}}/\mathfrak{p}R_{\mathfrak{p}}))$ with $\dim S - \dim R$.

(2) Given a cubic curve $y^2 = x^3 + bx + c$ with $b, c \in \mathbb{Z}$. Find a bound $M = M(b, c)$ on number of points of finite order.

(3) Let $\mathcal{C} \subset \mathbb{A}^2$ be a plane curve over an algebraically closed field. Show that one can resolve its singularities by blowing-ups.

(4) Let G be a finite group. A vector space over K is said to be representation of G is there is a group homomorphism $\rho : G \to GL(V)$. Two representation V, V' are equivalent if there is non-singular linear transformation $\phi : V \to V'$ compatible with ρ, ρ'. A representation is irreducible if V can not equivalent $V_1 \oplus V_2$ for some $V_1, V_2 \neq 0$. Describe all irreducible representation of the group S_3 over the field $\mathbb{C}$.