All rings are commutative with identity unless otherwise stated.

(1) Prove that \( \mathbb{C} \) is algebraically closed.
(2) Prove that the only division rings finite dimensional over \( \mathbb{R} \) are \( \mathbb{H}, \mathbb{C}, \mathbb{R} \).
(3) Let \( A \) be a domain. Show that the following are equivalent:
   (a) \( A \) is integrally closed.
   (b) \( A_p \) is integrally closed for all prime ideal \( p \).
   (c) \( A_m \) is integrally closed for all maximal ideal \( m \).

Now let \( A = k[x, y]/(y^2 - x^2(x - 1)) \), where \( k \) is algebraically closed. Find all maximal ideal \( m \) of \( A \) such that \( A_m \) is not integrally closed.

(4) Prove or disprove:
   (a) Let \( (A_m), (B, n) \) be local rings and \( m, n \) are the maximal ideals respectively. Let \( \varphi : A \to B \) be a non-zero homomorphism. Then \( \varphi^{-1}n = m \).
   (b) Let \( K \) be a field. For any integer \( n > 0 \) there is an irreducible polynomial of degree \( n \) in \( K[x] \).
   (c) Let \( \mathfrak{N} \) be the nil radical of \( R \). Then \( \mathfrak{N} = \cap_{p \in \text{Spec} R} p \).
   (d) Let \( I, J \) be \( p \)-primary ideals. Then there exist \( n \) such that \( I^n \subset J \).

(5) Let \( k \) be an algebraically closed field. And \( A = k[x, y, x] \). We consider an ideal \( I := (x^2 - yz, xz - x) \). Find a primary decomposition for \( I \).

(6) Let \( A \) be a local ring and \( M, N \) are finitely generated \( A \)-module. Prove that if \( M \otimes N = 0 \) then \( M = 0 \) or \( N = 0 \).

(7) Let \( R \) be a ring (not necessarily commutative) which is a finite dimensional algebra over a field. Show that \( R \) is Artinian.