

Advanced Algebra II

Homework 9

due on May. 14, 2004

- (1) (a) Let k be a field, show that the ring $k[x, y]/(xy - 1)$ and $k[x, y]/(y - x^2)$ are not isomorphic
(b) Assume furthermore that k is algebraically closed. Let $f \in k[x, y]$ be an irreducible quadratic polynomial. Show that $k[x, y]/(f)$ is isomorphic to either $k[x, y]/(y - x^2)$ or $k[x, y]/(xy - 1)$.
- (2) Let k be a field and R a finitely generated k -algebra. Prove that R is Artinian if and only if R is a finite k -algebra (i.e. to show that it's k -module of finite rank).
- (3) (a) Let $I, J \triangleleft R$ be ideals of R . Show that $\sqrt{I \cap J} = \sqrt{I} \cap \sqrt{J}$.
(b) If R is Noetherian, then show that a radical ideal has a decomposition into intersection of prime ideals.
- (4) Let $R[x]$ be a Noetherian ring. Does this imply that R is a Noetherian ring?
- (5) Let $\mathbb{C}\{x\} \subset \mathbb{C}[[x]]$ be the subring of convergent power series, that is, power series which is convergent in a neighborhood of 0. Is it Noetherian?