Advanced Algebra II

Homework 8

due on May. 7, 2004

(1) Let R, S be commutative rings with identity. And there is a ring homomorphism $f : R \to S$. Show that S can be viewed as an R-module. Moreover, show that $R[x] \otimes_R S \cong S$.
(2) Let G be a torsion group. Show that $G \otimes_{\mathbb{Z}} \mathbb{Q} = 0$. And show that $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q} \cong \mathbb{Q}$.
(3) We consider $\mathbb{R} \subset \mathbb{C} \subset \mathbb{H}$ in this problem. First show that $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \cong \mathbb{H}$ as vector space over $\mathbb{R}$. Is it possible to define a multiplication map on $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$ so that it's isomorphic to $\mathbb{H}$ as a ring?
(4) Let G be an abelian group, show that $G \otimes_{\mathbb{Z}} \mathbb{Z}_m \cong G/mG$. And show that $\mathbb{Z}_m \otimes_{\mathbb{Z}} \mathbb{Z}_n \cong \mathbb{Z}_(m, n)$.
(5) p.377, #1.
(6) p.377, #7.