

# Advanced Algebra II

## Homework 6

due on Apr. 23, 2004

- (1) Let  $M \cong \bigoplus_{i=1}^r M_i^{n_i}$  be a direct sum of simple modules  $M_i$ , where  $n_i$  denotes that  $M_i$  is repeated  $n_i$  times in the direct sum. Show that, up to isomorphism and permutation,  $M_i$  and  $n_i$  are uniquely determined.
- (2) Let  $D$  be a division ring,  $R \cong \bigoplus_{i=1}^r \text{Mat}_{n_i}(D)$ . Show that  $n_i$ s are uniquely determined, up to permutation.
- (3) Determine all left ideals of  $\text{Mat}_n(D)$ .
- (4) Let  $V$  be a 3-dimensional vector space over  $\mathbb{R}$ , and  $R := \text{End}_{\mathbb{R}}(V)$ . Determine subalgebra  $S$  of  $R$  such that  $V$  is a faithful simple  $S$ -module. Can you find any division ring in  $R$ ?
- (5) Pg. 424 # 4.
- (6) Pg. 424 # 5.
- (7) Pg. 424 # 7.