

Advanced Algebra II

Homework 4

due on Mar. 26, 2004

- (1) Complete the proof of Snake Lemma.
- (2) Give an example of a torsion free module which is not free.
- (3) Let M be a cyclic R module of order r , where R is a PID. Give an necessary and sufficient condition for $sM = M$ for a given $s \in R$.
- (4) Let V be a vector space over a field K . Fixed $A \in \text{End}(V) = \text{Hom}_K(V, V)$. We can consider V as a module over $K[x]$ by

$$\begin{aligned} K[x] \times V &\rightarrow V \\ (f(x), v) &\mapsto f(A)v. \end{aligned}$$

Suppose that W is a cyclic submodule of order $(x - \lambda)^n$. Show that there is a vector $w \in W$ such that $\{w, (x - \lambda)w, \dots, (x - \lambda)^{n-1}w\}$ is a basis of W .