

# Advanced Algebra II

## Homework 2

due on Mar. 12, 2004

- (1) Let  $L_1, L_2$  be two algebraically closed fields over  $K$ . Show there is an  $K$ -embedding either from  $L_1$  to  $L_2$  or from  $L_2$  to  $L_1$ .
- (2) A module is said to be cyclic if it's generated by one element. Show that the cyclic submodule generated by  $a$  is  $Ra + \mathbb{Z}a$ . And in case  $M$  is unitary, then the cyclic submodule generated by  $a$  is  $Ra$ .
- (3) Give an example of finitely generated  $R$ -module which is not a finitely generated abelian group.
- (4) Let  $f : M \rightarrow M$  such that  $f \circ f = f$ , then  $M = \text{Ker } f \oplus \text{Im } f$ .
- (5) Let  $f : M \rightarrow N$  and  $g : N \rightarrow M$  be  $R$ -homomorphism such that  $gf = \mathbf{1}_M$ , then  $N = \text{Im } f \oplus \text{Ker } g$ .