Advanced Algebra II

Homework 2

due on Mar. 5, 2004

(1) Show that the following are equivalent
  (a) every irreducible polynomial in $K[x]$ is separable.
  (b) every algebraic closure of K is Galois over K.
  (c) every algebraic extension over K is separable.
  (d) either $char(K) = 0$ or $char(K) = p$ and $K = K^p$.

  A field K satisfies above condition is said to be perfect.

(2) Show that a finite field is perfect, i.e. $F = F^p$.

(3) Verify that $tr.d.K(x_1, ..., x_n)/K = n$.

(4) Let K be a field. Let $f(x, y) \in K[x, y]$ be an irreducible polynomial. Let $R := K[x, y]/(f)$. Show that R is an integral domain.

  Let F be the quotient field of R. There is a natural embedding $K \hookrightarrow F$, therefore, for simplicity, we said that F is an extension over K. What is $tr.d.F/K$? Verify your answer.

(5) Let E_1, E_2 be intermediate fields of F/K. Show that
  (a) $tr.d.E_1E_2/K \geq tr.d.E_1/K$ for $i = 1, 2$.
  (b) $tr.d.E_1E_2/K \leq tr.d.E_1/K + tr.d.E_2/K$.

(6) Consider $K(x)/K$. Let $L \neq K$ be an intermediate field. Show that $L \cong K(t)$ for some t transcendental over K.