Advanced Algebra II

Homework 12

due on Jun. 4, 2004

(1) Let k be a field. Consider the inclusion $A := k[x^2 - 1] \subset B := k[x]$. It's clearly an integral extension. Let $\mathfrak{n} = (x - 1)$ be a maximal ideal of B. And let $\mathfrak{m} := \mathfrak{n} \cap A$. Describe $\mathfrak{m}$. Is $B_{\mathfrak{n}}$ integral over $A_{\mathfrak{m}}$?

(2) Let A be an integrally closed domain and K be its quotient field. Let $f(x) \in A[x]$ be a monic polynomial. If $f(x)$ is reducible in $K[x]$ then it's reducible in $A[x]$.
(Hint: consider roots of $f(x)$.)

(3) Let G be a finite subgroup of automorphism of a ring A. Let A^G denote the subring of G-invariants, i.e.

$$A^G := \{x \in A | \sigma(x) = x, \forall \sigma \in G\}.$$

Show that A is integral over A^G.

(4) Let $A \subset B$ be a finitely generated integral extension. Let $\mathfrak{p} \in \mathrm{Spec} A$. Show that there are only finitely many prime ideals lying over $\mathfrak{p}$.