

# Advanced Algebra II

## Homework 1

due on Feb. 27, 2004

In this homework, we assume that  $\text{char}(K) = p \neq 0$  unless otherwise stated.

- (1) Give an example of finite extension  $F/K$  which is neither separable nor purely inseparable.
- (2) Give an example of finite extension which is not a simple extension.
- (3) If  $a \in K - K^p$ , then  $x^{p^n} - a$  is irreducible for every  $n \geq 1$ .
- (4) Let  $F/K$  be an extension. Show that  $u \in F$  is separable over  $K$  if and only if  $K(u) = K(u^{p^n})$  for all  $n \geq 1$ .
- (5) Let  $F/K$  be an algebraic extension. Suppose that  $u \in S, v \in P$ . Show that  $K(u, v) = K(u + v)$ .
- (6) Examine the property of being purely inseparable under "extension", "lifting", and "compositum".