Advanced Algebra I

Homework 8

due on Nov. 24, 2006

(1) * Complete the uncompleted proof in the lecture.
(2) Determine the Galois group of $\mathbb{Q}[\sqrt[4]{5}, i]$ over $\mathbb{Q}$. Find all the subgroups and all the intermediate subfields. And also their correspondence.
(3) Let $\zeta = e^{\frac{2\pi i}{5}}$. Determine the Galois group of $\mathbb{Q}[\zeta]$ over $\mathbb{Q}$. Find all the intermediate subfields. Solve ζ by radicals.
*Can you do it for $\zeta = e^{\frac{2\pi i}{17}}$? This also shows that regular 17-gon is constructible. Can you work it out?
(4) Let F/K be an extension. If $u_1, ..., u_n \in F$ is separable over K, then $K(u_1, ..., u_n)$ is separable over K.
(5) Let F/K be an extension with an intermediate field E. Assume that $E = K(u_1, ..., u_r)$, where u_i are some roots of $f(x) \in K[x]$. Then F is a splitting field of $f(x)$ over K if and only if $f(x)$ is a splitting field of $f(x)$ over E.
(6) Given an algebraic extension F/K, we may define the normal closure of F over K to be the smallest $N \subset \overline{F} = \overline{K}$, normal over K and containing F.
Determine the normal closure of $\mathbb{Q}[\sqrt{2} + \sqrt{3}]$ over $\mathbb{Q}$.