Advanced Algebra I

Homework 7

due on Nov. 17, 2006

(1) * Complete the uncompleted proof in the lecture.

(2) Let F/K be an algebraic extension. Let $u, v \in F$ with minimal polynomial $f(x), g(x)$ respectively. If $\deg(f(x))$ and $\deg(g(x))$ are relatively prime, then $g(x)$ is irreducible in $K(u)[x]$.

(3) Study the irreducibility of $x^5 + x + 1$ in $\mathbb{Q}[x]$.

(4) * Let F/K be an algebraic extension. Let $u, v \in F$ with minimal polynomial $f(x), g(x)$ respectively. If there any systematical way to construct a polynomial having $a + b$ as a root. And for ab?

(Hint: Use resultant).

(5) Give an example of an irreducible polynomial $p(x) \in \mathbb{Q}[x]$ such that $p(x^2)$ is reducible in $\mathbb{Q}[x]$.

(6) F is an algebraic closure of K if and only if F is algebraic over K and for every algebraic extension E of K, there exists a injective K-homomorphism $E \to F$.