(1) * Complete the uncompleted proof in the lecture.
(2) Construct a field $F$ of 9 elements. Then $F^*$ is a cyclic group of 8 elements. Find a generator of $F^*$.
* How about fields of $3^n$ elements?
(3) Let $F = \mathbb{Q}(\sqrt{3}, i, \omega)$, where $\omega = \frac{-1 + \sqrt{3}i}{2}$. Find $[F : \mathbb{Q}]$ and a basis of $F$ over $\mathbb{Q}$.
(4) Let $F = \mathbb{Q}(\sqrt[3]{2}, \omega)$. Find $[F : \mathbb{Q}]$ and a basis of $F$ over $\mathbb{Q}$.
Moreover, find an element $u$ such that $F = \mathbb{Q}(u)$.
(5) Verify Proposition 3.27.
(6) In the field $K(x)$ we consider $u = \frac{x^4 + x^2 + 1}{x + 1}$. What is $[K(x) : K(u)]$? In general, if $u = \frac{f(x)}{g(x)}$, then what is $[K(x) : K(u)]$?
(7) Let $\Phi_p(x) := \frac{x^p - 1}{x - 1} = x^{p-1} + \ldots + 1 \in \mathbb{Q}[x]$. Show that $\Phi_p(x)$ is irreducible.