Advanced Algebra I

Homework 6

due on Nov. 10, 2006

(1) * Complete the uncompleted proof in the lecture.

(2) Construct a field F of 9 elements. Then F^* is a cyclic group of 8 elements. Find a generator of F^*.
* How about fields of 3^n elements?

(3) Let $F = \mathbb{Q}(\sqrt{3}, i, \omega)$, where $\omega = \frac{-1+\sqrt{3}i}{2}$. Find $[F : \mathbb{Q}]$ and a basis of F over $\mathbb{Q}$.

(4) Let $F = \mathbb{Q}(\sqrt[3]{2}, \omega)$. Find $[F : \mathbb{Q}]$ and a basis of F over $\mathbb{Q}$. Moreover, find an element u such that $F = \mathbb{Q}(u)$.

(5) Verify Proposition 3.27.

(6) In the field $K(x)$ we consider $u = \frac{x^4+x^2+1}{x+1}$ What is $[K(x) : K(u)]$? In general, if $u = \frac{f(x)}{g(x)}$, then what is $[K(x) : K(u)]$?

(7) Let $\Phi_p(x) := \frac{x^p-1}{x-1} = x^{p-1} + ... + 1 \in \mathbb{Q}[x]$. Show that $\Phi_p(x)$ is irreducible.