

Advanced Algebra I

Homework 4

due on Oct. 27, 2006

- (1) * Complete the uncompleted proof in the lecture.
- (2) Every subgroup and every quotient of a nilpotent group is nilpotent.
- (3) Let G be a subgroup of S_7 generated by (1234567) and $(34)(26)$. Show that $|G| = 168$.
*One can even show that G is simple.
- (4) If $N \triangleleft G$ and $N \cap G' = \{e\}$, then $N < Z(G)$.
- (5) (a) If p, q are prime. then every group of order p^2q is solvable.
(b) If p, q are prime with $p < q$, then every group of order pq^n is solvable.
- (6) (a) A finite group of order < 60 is non-simple.
(b) A finite group of order < 60 is solvable.
- (7) (a) Given a solvable group G , there is a subgroup $H < G$ such that $[G : H]$ is prime.
(b) Moreover, if H is a maximal proper subgroup of a finite solvable group G , then $[G : H]$ is a prime power.