Advanced Algebra I

Homework 3

due on Oct. 20, 2006

(1) * Complete the uncompleted proof in the lecture.
(2) Let $N, K < G$ such that $N \lhd G$ and $NK = G$, $N \cap K = \{e\}$. Can you realize G as a semidirect product of N and K?
(3) For every $d|24$, there is a subgroup of S_4 of order d. Moreover, if $d \neq 4$, then any two subgroup or order d are isomorphic.
(4) Classify groups of order 28 up to isomorphism.
(5) Let p be a prime. Describe the maximal p-subgroup of $\mathbb{Q}/\mathbb{Z}$.
(6) Show that the rigid motion of $\mathbb{R}^n$ preserving the origin is $O(n, \mathbb{R})$.
(7) A_5 is simple, i.e. A_5 has no normal subgroup. A simple group of order 60 must isomorphic to A_5.
(8) In $\mathbb{Z}_{p^2} \oplus \mathbb{Z}_{p^3}$, how many elements of order p, p^2, p^3?
(9) Let G, H, K be finitely generated abelian groups.
    (a) If $G \oplus G \cong H \oplus H$, then $G \cong H$.
    (b) If $G \oplus H \cong K \oplus H$, then $G \cong K$.
(10) *Read Artin's chapter 5 and classify 17 crystallographic groups.