Advanced Algebra I
Homework 2
due on Oct. 6, 2006

(1) * Complete the uncompleted proof in the lecture.

(2) Determine the structure of Sylow 2-subgroup of S_5.

(3) There is no non-abelian simple group of order < 60.

(4) Show that the only subgroup of order 12 in S_4 is A_4.

(5) Show that the group T of order 12 is a semidirect product of $\mathbb{Z}_3$ and $\mathbb{Z}_4$.

(6) Show that A_4, D_{12}, T are not isomorphic to each other.

(7) * For $n \geq 3$, we consider a group of order 2^n, $Q = <a, b | a^{2^{n-1}} = 1, bab^{-1} = a^{-1}, b^2 = a^{2^{n-2}} >$.

   (a) if G is a group of order 2^n which is generated by x, y such that $x^{2^{n-2}} = y^2 = (xy)^2$, then $G \cong Q$.

   (b) there is a unique element z of order 2 in Q.

   (c) $Z(Q) = <z>$.

   (d) $Q/Z(Q) \cong D_{2^{n-1}}$.

   (e) Q is isomorphic to the group in Example 2.5.6.

(8) Let $G = SL(2, \mathbb{F}_5)$ the multiplication group of 2×2 matrices with coeffcient in $\mathbb{F}_5$ with determinant 1.

   (a) Show that $|G| = 120$.

   (b) Determine the structure of Sylow 2-subgroup of G.

   (c) Show that $G \not\cong S_5$.