Advanced Algebra I

Homework 15

due on Jan. 19, 2007

(1) Complete the proof of Theorem 4.5.6 and 4.5.7
(2) Let $F : \mathcal{A} \to \mathcal{B}$ be a left exact functor and $\mathcal{A}$ has enough injectives. Show that $R^iF(A) := H^i(F(I^\bullet))$ is well-defined. That is, independent of choice of injective resolution $I^\bullet$.
(3) Given a complex $K^\bullet$, construct an injective resolution $I^\bullet$ of $K^\bullet$. That is, a quasi-isomorphism $f : K^\bullet \to I^\bullet$.
(4) In the category of abelian groups, show that an injective object is a divisible group.
(5) We can define projective in a similar way (with arrow reversing). That is for any exact sequence $B \xrightarrow{\alpha} C \to 0$ and $f : P \to C$, there exist $g : P \to B$ such that $g\alpha = f$.

    Show that for any exact sequence $0 \to A \to B \to P \to 0$ with P being projective, the sequence splits.

    We now consider the category of abelian groups Ab. Determine the projective objects. (Hint: every abelian group is a quotient of free abelian group.)