

# Advanced Algebra I

## Homework 13

due on Jan. 5, 2007

- (1) Complete the uncompleted proof in the lecture. Especially  $tr.deg.KF = tr.deg.KE + tr.deg.EF$ .
- (2) Determine the transcendental degree of  $F/K$  for the following  $F$ :
  - (a)  $F$  is the quotient field of  $K[x, y]/(y^2 - x^3 - x)$ .
  - (b)  $F$  is the quotient field of  $K[x, y, z, w]/(xz - y^2, xw - yz)$
- (3) Compute the homology groups of torus  $T^2 = S^1 \times S^1$ ,  $S^n$ , and Klein bottle.
- (4) Compute the homology groups of  $\mathbb{R}^2$  and  $\mathbb{R}^2 - \{0\}$ .
- (5) Consider the subring  $K[t^n, t^m] \subset K[t]$  with  $(m, n) = 1$  and let  $F$  be its quotient field. Show that  $F = K(t)$ .
- (6) Prove that  $\mathbb{C}$  have infinitely many automorphisms.
- (7) Suppose that we have extension  $K \subset E \subset F$ . If  $F/K$  is finitely generated, then so is  $E/K$ .