Advanced Algebra I

Homework 1

due on Sep. 29, 2006

(1) Complete the uncompleted proof in the lecture.
(2) Let A be an infinite set and for $i = 1, 2, ...$, $|B_i| \leq |A|$. Then $| \amalg_{i=1,2,...} B_i| \leq |A|$.
(3) Let A be an infinite set and Σ be the set of finite subsets of A. Show that $|Sigma| = |A|$.
(4) Let F be an infinite field. Then $|F| = |F[x]| = |F(x)|$.
(5) Let N, K be subgroups of G. If $xy = yx$ for all $x \in N, y \in K$ and $NK = G$, $N \cap K = \{e\}$. Then $G \cong N \times K$.
(6) Let V be a n-dimensional vector space over $\mathbb{R}$. Let $GL(V)$ denoted the set of invertible linear transformation from V to V. It's clear that $GL(V)$ is a group under the composition. (One may think it as $GL(n, \mathbb{R})$, the groups of invertible $n \times n$ matrices.) Let $SO(n, \mathbb{R}) := \{A \in GL(n, \mathbb{R}) | AA^t = I\}$. Show that $SO(n, \mathbb{R})$ is a subgroup. Is it normal?
(7) Let T, D, I be the subgroups of $SO(3, \mathbb{R})$ that preserves regular tetrahedron, octahedron, icosahedron respectively. Determine their cardinality. Can you say something about the structure of these groups?