

# Introduction to Algebraic Geometry

## Homework 5

Discuss on Dec. 9, 2005. Due on Dec. 16, 2005

Now  $k$  denotes a field which is algebraically closed.

- (1) Describe the affine scheme, including points and structure sheaves, of the following  $\mathbb{C}[x]/(x^2)$ ,  $\mathbb{C}[x]/(x^3 - x^2)$ ,  $\mathbb{C}[x]/(x^2 - x)$ .
- (2) Consider the ring homomorphism  $k[x] \hookrightarrow k[x, y]$  and natural surjection  $k[x, y] \rightarrow k[x]$ . Describe the corresponding morphism of schemes, including closed sets, distinguished open sets and structure sheaves.
- (3) Let  $\varphi : \mathcal{F} \rightarrow \mathcal{G}$  be a morphism of presheaves. Show that  $\ker(\varphi)$ ,  $\operatorname{coker}(\varphi)$  and  $\operatorname{im}(\varphi)$  are presheaves. Also show that  $\ker(\varphi)_x = \ker(\varphi_x)$ , similarly for  $\operatorname{coker}(\varphi)$  and  $\operatorname{im}(\varphi)$ .
- (4) Let  $R_1 = k[x, y]/(y - x^3)$ ,  $R_2 = k[x, y]/(y^2 - x^3)$ , determine  $\operatorname{Hom}_k(R_i, k[t]/(t^{n+1}))$  for  $n = 0, 1, 2$  and  $i = 1, 2$ .