Introduction to Algebraic Geometry
Homework 5
Discuss on Dec. 9, 2005. Due on Dec. 16, 2005

Now $k$ denotes a field which is algebraically closed.

(1) Describe the affine scheme, including points and structure sheaves, of the following $\mathbb{C}[x]/(x^2)$, $\mathbb{C}[x]/(x^3 - x^2)$, $\mathbb{C}[x]/(x^2 - x)$.

(2) Consider the ring homomorphism $k[x] \hookrightarrow k[x, y]$ and natural surjection $k[x, y] \twoheadrightarrow k[x]$. Describe the corresponding morphism of schemes, including closed sets, distinguished open sets and structure sheaves.

(3) Let $\varphi : \mathcal{F} \rightarrow \mathcal{G}$ be a morphism of presheaves. Show that $\ker(\varphi), \text{coker}(\varphi)$ and $\text{im}(\varphi)$ are presheaves. Also show that $\ker(\varphi)_x = \ker(\varphi_x)$, similarly for $\text{coker}(\varphi)$ and $\text{im}(\varphi)$.

(4) Let $R_1 = k[x, y]/(y - x^3), R_2 = k[x, y]/(y^2 - x^3)$, determine $\text{Hom}_k(R_i, k[t]/(t^{n+1}))$ for $n = 0, 1, 2$ and $i = 1.2.$