Riemann Surface

Homework 2

Oct. 8, 2004. Due on Oct. 15, 2004

(1) Let X be a compact Riemann surface and f a meromorphic function on X. Let $z_1, .., z_t$ be zeros of f with order $n_1, ..., n_t$ respectively and $p_1, ..., p_s$ be poles with order $m_1, ..., m_s$ respectively. Show that

$$\sum_{i=1}^{t} n_i = \sum_{j=1}^{s} m_j.$$

Moreover, consider $\tilde{f} : X \to \mathbb{P}^1$ the induced holomorphic map. Show that for general point $y \in \mathbb{P}^1$, the preimage $\tilde{f}^{-1}(y)$ consists of d points, where $d := \sum_{i=1}^{t} n_i$.

(2) Let $X := \mathbb{C}/\Gamma$ be a complex torus, and $\mathcal{P}(z)$ be the Weierstrass $\mathcal{P}$-function. Determine the zero of $\mathcal{P}'(z)$?

(3) Let X be a compact Riemann surface. Given two distinct points p, q, we consider $\mathbb{C}(p)$ and $\mathbb{C}(q)$ the skyscraper sheaf at p, q respectively. There is a natural map $\varphi : \mathcal{O} \to \mathbb{C}(p) \oplus \mathbb{C}(q)$. Let $\mathcal{I}_{p,q} := \ker \varphi$ And verify that $H^0(X, \mathcal{O}) \to H^0(X, \mathbb{C}(p) \oplus \mathbb{C}(q))$ is not surjective and $H^1(X, \mathcal{I}_{p,q}) \neq 0$.

(4) Let $\varphi : \mathcal{F} \to \mathcal{G}$ be a morphism of sheaves. Show that $(\ker \varphi)_p = \ker \varphi_p$ and $(\mathrm{im}\varphi)_p = \mathrm{im}\varphi_p$.

(5) Let $\mathcal{F}, \mathcal{G}$ be sheaves on X. For any open set $U \subset X$, we associate an abelian group $Hom(\mathcal{F}|_U, \mathcal{G}|_U)$. Show that such assignment gives a presheaf, denoted $\mathcal{H}om(\mathcal{F}, \mathcal{G})$. Show furthermore that this is indeed a sheaf.