Advanced Algebra I

Homework 3

due on Oct.17, 2003

Part A.

(1) Find a example of a solvable group which is not nilpotent.
(2) Let G be a nilpotent group. Show that every subgroups and every quotient groups are nilpotent.
(3) Let G be a nilpotent group of order n. Prove that for $m|n$, there is a subgroup of order m in G.
(4) Let G be a finite solvable group. Show that G has a solvable series such that each factor is a cyclic group of prime order. (Hint: you might need the fundamental theorem for finite abelian group).

Part B.

(1) Show that a group of order p^2q , where p, q are distinct primes, is solvable.
(2) Show that a simple group of order 60 is isomorphic to A_5. (Hint: Consider the action $G \times G/H \to G/H$ which gives a homomorphims $\tilde{\alpha} : G \to S_{|G/H|}$. Show that this is an injection. And show that there is a subgroup H of index 5.)
(3) If H, K are solvable subgroups of G, and $H \lhd G$, then HK is a solvable subgroup of G.
(4) Let G be a finite group and let $N \lhd G$ be a normal subgroup such that $(|N|, |G/N|) = 1$
   (a) Let H be a subgroup that $|H| = |G/N|$. Prove that $G = HN$.
   (b) Let $\sigma Aut(G)$ be an automorphism. Prove that $\sigma(N) = N$.