

Advanced Algebra I

Homework 2

due on Oct.3, 2003

Part A.

- (1) Find the Sylow 2-subgroup and 3-subgroup of S_4, S_6
- (2) Let G be a group. Show that $Aut(G)$, the set of automorphism, is a group by composition. Furthermore, consider the conjugation $G \times G \rightarrow G$, we have and group homomorphism $\tilde{\alpha} : G \rightarrow Aut(G)$. What is kernal of $\tilde{\alpha}$?
- (3) Show that group of order p^2 is abelian.
- (4) Show that D_6 is not isomorphic to A_4 . (They both are non-abelian group of order 12).

Part B.

- (1) Show that $Z(S_n) = \{e\}$. And show that $Aut(S_4) \cong S_4$.
- (2) Let p be a odd prime. Prove that there are at most two non-abelian group of order p^3 .