

Advanced Algebra I

Homework 12

due on Dec. 19, 2003

- (1) Determine the Galois group of the following extension:
 - (a) $\mathbb{Q}(\sqrt{3}, \sqrt{5})$ over \mathbb{Q} .
 - (b) Let F be the splitting field of $x^5 - 2$ over \mathbb{Q} . What is the Galois group $\text{Gal}_{F/\mathbb{Q}}$?
- (2) Show that if $[F : K] = 2$ then F is normal over K .
- (3) Prove that an algebraically closed field is infinite.
- (4) Let $K = \overline{K}$ be an algebraically field. Let $f(x_1, \dots, x_n) \neq 0 \in K[x_1, \dots, x_n]$. Prove that there are $a_1, \dots, a_n \in K$ such that $f(a_1, \dots, a_n) = 0$. That is $f = 0$ has a solution in K^n . What if K is not algebraically closed?
- (5) Let F be a finite field of p^n elements and P be its prime field, that is, a subfield of p elements.
 - (a) Consider $\sigma : F \rightarrow F$ by $\sigma(u) = u^p$. Show that $\sigma \in \text{Gal}_{F/P}$.
 - (b) Determine $\text{Gal}_{F/P}$