

Advanced Algebra I

Homework 1

Part A.

- (1) Give an example of poset such that (and verify your example)
 - (a) it's not totally ordered.
 - (b) it has a maximal element but no minimal element.
- (2) Let G be a group acting on X .
 - (a) Let $x, y \in X$ such that $y = gx$ for some $g \in G$. What is the relation between G_x and G_y ? Prove that $|G_x| = |G_y|$.
 - (b) Show that each of the orbit is a transitive G -set.
- (3) Let F be a field such that $|F|$ is infinite. Show that $|F[x]| = |F|$. What is $|F[x]|$ if F is finite?

Part B.

- (1) Suppose that $n > 5$. Show that S_n has no subgroup of index t for $2 < t < n$.
- (2) Let $\alpha : G \times X \rightarrow X$ be a group action, and let $\tilde{\alpha} : G \rightarrow S_X$ be the induced representation.
 - (a) If $K = \ker \tilde{\alpha}$, then show that G/K acts on X in a natural way.
 - (b) If X is a transitive G -set, then $|\ker \tilde{\alpha}| \leq |G|/|X|$.