Advanced Statistical Inference I

Homework 1: Probability Theory

Due Date: October 7th

1. (Detect mixture distribution) Exercise 1.6.
2. (Countable additivity and Kolmogorov's Axiom) Exercise 1.12 and Exercise 1.35.
3. (Information and Conditioning) Exercise 1.32 and Exercise 1.37.
4. (Is there a cheating?) Exercise 1.22.
5. (Sampling and central limit theorem)
   (a) Exercise 1.28.
   (b) Exercise 1.31.
6. (Model time series and etc)
   (a) Exercise 1.38.
   (b) US weather service has a record for each January from 1948 to 1983 at the station of Snoqualmie falls. According to the definition of raining day, there are 325 sunny days and 791 rainy days. Someone proposes a model on p_i, which is the probability of a particular day, by a coin-tossing model in which $p_i = p$ and the outcomes are independent. Under this modeling, derive the probability of getting 325 sunny days in 1116 days.
   (c) (Continuation of (b)) What kind of p will maximize the occurring of such an event? (Here the event refers to 325 sunny days in 1116 days.)
   (d) Another person proposes an alternative model in which p_i will depend on p_{i-1}. In particular,
   $$p_i = \begin{cases} p_w & \text{if the } (i-1)\text{th day is rainning} \\ p_d & \text{if the } (i-1)\text{th day is not rainning} \end{cases}$$
   The available data for this modeling is as follows:

	Today is sunny	Today is rainy	Total
Yesterday is sunny	189	123	309
Yesterday is rainy	128	643	771
Total	314	766	1080

   Please write down the probability of getting 791 rainy days in 1116 days.
7. (What is the meaning of random assignment? Think of lottery.)
   (a) Exercise 1.20 and Exercise 1.46.
   (b) What is the probability distribution of X_1?
   (c) If someone does this experiment once, you are asked to guess the outcome of X_1 defined in Exercise 1.46. Let Y_j denote the rule that someone will make a guess j. If the guess matches the outcome of X_1, a prize of 100 dollars will be given. Otherwise, there is no reward. Determine the expected return of rule Y_j and decide which Y_j gives the highest return.
8. (Comparison of two random variables)
   (a) Exercise 1.49.

(b) Let X and Y be two random variables defined as in Example 1.5.4 with p_X and p_Y, respectively. Here p_X and p_Y refer to the probability of a head on any given toss for those two coins, respectively. When $p_X > p_Y$, is F_Y is stochastically greater than F_X? Give reason to support your conclusion.