Financial Time Series I and Methods of Statistical Prediction

Homework 3: Review on Basics

Due Date: 12/05/2002

1. Let $X_1, \ldots, X_n$ be iid with density $f(x;\beta) = \beta \exp(-\beta x)$ For $x > 0$ and $\beta > 0$. Derive the likelihood ratio test of size α for $H_0 : \beta = \beta_0$ versus $H_1 : \beta \neq \beta_0$ and show that the rejection region is of the form $\{\bar{X}\exp(-\beta_0 \bar{X}) \leq c\}$.
2. (continuation of problem 1) Suppose we wish to test $H_0 : \beta = 1$ versus $H_1 : \beta \neq 1$ based on the following data:
$$0.584, 2.611, 0.287, 0.968, 1.933, 2.099, 1.876, 0.882, 0.695, 0.116, 1.926,$$
$$2.343, 2.059, 1.183, 0.184, 1.85, 0.233, 0.01, 0.061, 0.397$$
(a) Use the test derived in problem 1 with size 0.05.
(b) In problem 1, we rely on the asymptotic approximation result to check whether H_0 should be rejected. Use Monte Carlo method to derive approximation of distribution of test statistic derived from likelihood ratio test.
3. In a traffic study the average speed of cars (in miles per hour) and the *acceleration noise* level were measured for each of 30 sections of roadway. Let x denote the average speed and y denote the noise level. They are $10.0, 11.5, 12.0, 18.0, 19.5, 20.0, 28.0, 31.0, 32.0, 33.0, 33.5, 35.0, 36.0, 39.0, 40.0, 40.5, 41.5, 42.0, 42.5, 43.0, 43.9, 44.0, 44.1, 45.0, 47.9, 48.0, 50.0, 50.2, 50.5, 51.0$ and $1.60, 1.45, 1.10, 1.05, 1.25, 0.50, 0.70, 0.45, 0.40, 0.10, 0.18, 0.11, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.40, 0.20, 0.06, 0.09, 0.35, 0.38, 0.37, 0.10, 0.12, 0.25$, respectively.
(a) Use R to plot noise level y against average speed x with x-label, y-label, and title being *average speed*, *noise level*, and *Study on Association of Noise Level and Average Speed*. (b) Calculate the correlation of noise level with speed. Use the nonparametric bootstrap method to derive a 95% confidence interval of this correlation coefficient.
4. Generate samples of size 25, 50, and 100 from a normal distribution. Construct normal probability plots. Do this several times to get an idea of how probability plot behave when the Underlying distribution is really normal.
(a) Repeat the above for a chi-square distribution with 10 df.
(b) Repeat the above for $Y = Z/U$, where $Z \sim N(0,1)$ and $U \sim U[0,1]$ And Z and U are independent.
(c) Repeat the above for a uniform distribution. (d) Repeat the above for an exponential distribution. (e) Can you distinguish between the normal distribution and subsequent nonnormal distributions? (f) Suppose that a sample is taken from a symmetric distribution whose tails decrease more slowly than those of the normal distribution. What would be the qualitative shape of a normal probability plot of this sample? You can try the Cauchy distribution with probability density function $\pi^{-1}(1+x^2)^{-1}$.
5. Prove this version of the Bonferroni Inequality:
$$P(\cap_{i=1}^{n} A_i) \geq 1 - \sum_{i=1}^{n} P(A_i^c).$$

In the context of simultaneous confidence intervals, what is A_i and what is A_i^c?