Financial Time Series I and Methods of Statistical Prediction

Homework 2: Review on Basics

Due Date: 11/01/2002

1. A company has 11 mathematicians on its staff, of whom 3 are women. The president of the company is concerned about the small number of women mathematicians. The president learns that about 40% of the mathematicians in the United States are women, and asks you to investigate whether or not the number of women mathematicians in the company is consistent with the national pool.
(a) Find the probability that there are 3 or fewer women in a randomly selected group of 11 mathematicians.
(b) The president knows very little about statistics. Instead, the president asks you to use simulation to estimate the probability that there are 3 or fewer women in a randomly selected group of 11 mathematicians.

2. The table shown below provides information about the scores on the 1997 and 1998 AP Statistics Examinations. The data for 1997 include all students who took the exam in 1997. The data for 1998 were obtained from a random sample of 200 students who took the exam in 1998.

		All Students from 1997		Sample from 1998	
	Exam Grade	Number of Students	Percent	Number of Students	Percent
Extremely well qualified	5	1193	15.6	21	10.5
Well qualified	4	1692	22.1	58	29
Qualified	3	1872	24.5	46	23
Possibly qualified	2	1502	19.6	45	22.5
No Recommendation	1	1387	18.1	30	15
Total Number of Students		7646		200	
Mean Grade		2.97		2.975	
Standard Deviation		1.33		1.242	

You will use the data from this sample to describe how you expect the performance of all students who took the exam in 1998 to differ from the performance of all students who took the exam in 1997.
(a) Do these data provide evidence of a change in the distribution of grades from 1997 to 1998? Give appropriate statistical justification to support your conclusion.
(b) Is there evidence that there was a change in the mean grade from 1997 to 1998? Give appropriate statistical evidence to support your conclusion.
(c) Using the evidence from this sample, describe how you expect the performance of all students who took the exam in 1998 to differ from the performance of all students who took the exam in 1997.

3. Studies find a negative correlation between hours spent watching television and scores on reading tests. Does watching television make people less able to read? Discuss briefly.

4. Suppose men always married women who were exactly 8% shorter. What would the correlation between their heights be?

5. Suppose 15 rats are used in a biomedical study where the rats are injected With cancer cells and given a cancer drug that is designed to increase their survival rate. The survival times, in months, are $14, 17, 27, 18, 12, 8, 22, 13, 19$ and 12. Assume that the exponential distribution applies.
   (a) Give a maximum likelihood estimate of mean survival.
   (b) Give an estimate of mean survival based on the method of moments.

6. A gasoline company tested 20 one-gallon samples of gasoline produced during a day. The results were as follows: $87.5, 86.9, 86.6,$ $87.3, 87.9, 88.0,$ $86.7, 87.5, 87.2,$ $87.0, 88.1, 87.5,$ $86.5, 87.7, 88.0,$ $87.1, 87.0, 87.6,$ $87.5, 88.3$.
   (a) Give a 95% confidence interval for the mean octane rating of the day's production. Assume a normal population.
   (b) Construct a normal probability plot for this data to investigate whether the assumption of normality used in part (a) seems plausible. What do you conclude? (Note: This is a relatively small sample - of size n = 20 - so any conclusion concerning normality of the population must necessarily be a very tentative one.)

7. Suppose the gasoline company in the above problem performed similar tests on each of 100 days, and thus produced 100 confidence intervals - one for each of the 100 days. Let Y denote the number of intervals out of the 100 that contain the true mean octane Rating of that day's production.
   (a) What is the distribution of Y?
   (b) What is the probability that $Y \geq 90$? $Y \geq 95$? (a normal approximation that gives a good approximation to this answer would also provide an acceptable solution to this question.)

8. Let $X \sim Gamma(\alpha, \beta)$. In other words, $f(x) = \frac{1}{\Gamma(\alpha)} \frac{1}{\beta^\alpha} X^{\alpha-1} \exp(-x/\beta)$, $x > 0$, $\alpha > 0$, $\beta > 0$. Show that $Var(X) = \alpha\beta^2$.

9. If the random variable X follows the normal distribution with $\mu = 0$, $\sigma^2 = 1$ and $Y = \exp(X)$, find the probability density of Y. This is called the lognormal distribution.