Financial Time Series

Project 2: Test for Unit Root and ARFIMA

Due Date: May 23rd

In this project, there are three parts. In the first part, we simulate the limiting distributions $T(\hat{\phi}_T - 1)$ given in (3.2) of the textbook and $(T/s_T)(\hat{\phi}_T - 1)(T^{-2}\sum_{t=1}^T x_{t-1}^2)^{1/2}$ in page 68. In the second part, we will use simulation to get some feeling on the decay of autocorrelations. In the third part, we look at some real data sets to get some idea on long memory. In addition, try out the technique of data transformation.

Part I:

Problem 1: Study the limiting distribution $T(\hat{\phi}_T - 1)$ according to the suggestion given in page 69 of the textbook. Consider $\sigma^2 = 1$, $T = 1000$, and $n = 25000$ as in the book. Your result should include something similar to Figure 3.1.

I write my own program for this project and find that the following commands are useful.

- cumsum: It can be used to calculate $\sum_{s=0}^{t-1} a_s$ quickly.
- *for* and *For* are two possible ways to do repeating work.
- Use help(arithmetic) to find out efficient ways to calculate $\sum_{t=1}^T (\sum_{s=0}^{t-1} a_s) a_t$ and $\sum_{t=1}^T (\sum_{s=0}^{t-1} a_s)^2$.
- density: It can be used to get a smooth estimate of density function.

Sample program

- Step 1: Write a function to calculate

$$\frac{T\sum_{t=1}^T (\sum_{s=0}^{t-1} a_s) a_t}{\sum_{t=1}^T (\sum_{s=0}^{t-1} a_s)^2}.$$

  - Generate T+1 pseudo-random $N(0,1)$ variates
  - Use cumsum and * to calculate the last formula in page 69.
  - The function is

```
simu1<-function(T0){
T1<-T0+1; a<-rnorm(T1); acum<-cumsum(a)
denominator<-sum(acum*acum); a1<-a[2:T1]; acum1<-acum[1:T0]
numerator<-T1*sum(a1*acum1); numerator/denominator
}
```

- Step 2: Create a vector (I use result) to store results from the simulation.
  My program is
  T0< − 1000; nsimu < − 25000; result¡- 1:nsimu
  Remark. Suppose we will do 1000 simulations and the outcome from each simulation is a vector of length 3. We can specify a matrix to store the results from the ith simulation experiment in the ith row of that matrix. A possible command is as follows:
  $$result < -matrix(1:(1000*3), ncol = 3)$$
- Step 3: Use *For* instead of *for* to carry put the simulation to increase the efficiency of program.
  I use the following command but I urge you to try *For*.
  $$for\ (i \text{ in } 1:nsimu)\ \ result[i] < -simu1(T0)$$
- Step 4: Use *density* to get a smooth estimate of limiting distribution. I use the following command but I urge you to try *gaussian kernel*.
  $$plot(density(result), type = "l")$$

Problem 2: We study limiting distribution $(T/s_T)(\hat{\phi}_T - 1)(T^{-2}\sum_{t=1}^{T} x_{t-1}^2)^{1/2}$ Your result should include something similar to Figure 3.2. (You don't need to do the normal curve.)

Part II:

Problem 3. Simulate four $AR(2)$ processes as specified in Figure 2.4 and find their corresponding SACF and PACF. Your answer should include something similar to Figures 2.3 and 2.4.
Problem 4. Simulate two $MA(2)$ processes as specified in Figure 2.5 and find their corresponding SACF and PACF.
Problem 5. Repeat Problem 3 but consider $ARFIMA(2, 0.33, 0)$.
Problem 6. Repeat Problem 4 but consider $ARFIMA(0, 0.33, 2)$.

Part III:

Problem 7. To do this problem, you first need to download the data from the course web. In this data set, it gives daily log returns of Hewlett-Packard, value-weighted, equal-weighted and SP500 index those four series. To check for long-range dependence, compute the first 100 lags of ACF of the absolute daily returns of those four series. Is there evidence of long-range dependence? Why? Note that long memory means ACF decays slowly.
Repeat the above analysis on the squared daily returns of those four series. Is there evidence of long-range dependence? Why?