

Financial Time Series

Project 1: Linear time series analysis

Due Date: Part I: April 18th; Part II: April 25th

In the first part of this project, Splus programming language will be used to repeat all analysis is done in Example 2.2 of the textbook. In particular, we are doing model building and model checking. In the second part of this project, we use simulation to evaluate the validity of those ideas on model building and model checking.

Part I: Analysis in Example 2.2

Problem 1: Ex. 2.2 considers the problem of modeling the UK interest rate spread. It studies the spread between 20 Year UK Gilts and 91 day Treasury Bills based on 526 monthly observations for the period from 1952 to 1995.

1. Construct the time series plot as in Figure 2.7.
2. Reproduce Table 2.2 in which gives the SACF and SPACF up to $k = 12$.
3. Based on SACF and SPACF, it proposes to use an $AR(2)$ process to model this time series. Give reasons to justify such a proposal.
4. In the book, it uses OLS regression technique to find the following fitted $AR(2)$ model

$$x_t = 0.045(\pm 0.023) + 1.182(\pm 0.043)x_{t-1} - 0.219(\pm 0.043)x_{t-2} + a_t$$

with $\hat{\sigma} = 0.448$. Give a brief explanation why OLS can be used to get the estimates of unknown parameters. Carry out this analysis.

Problem 2: Check whether the $AR(2)$ is a reasonable model?

1. If $AR(2)$ is a reasonable model, it is expected that the residuals \hat{a}_t behave as white noise. Here $\hat{a}_t = x_t - 0.045 - 1.182x_{t-1} + 0.219x_{t-2}$. Give the definition of white noise. Propose methods for checking whether \hat{a}_t behave as white noise. The analysis should include time series plot of \hat{a}_t , calculate its SACF and SPACF, and describe its distribution.
2. If $AR(2)$ is a reasonable model, it is expected that a bigger model will not improve the fit. This idea is implemented by using $AR(3)$ process and $ARMA(2, 1)$ process to test model adequacy. Repeat the analysis done in the book to confirm the adequacy an $AR(2)$ process.

Part II:

Use simulation to evaluate the validity of those ideas on model building and model checking

Problem 3: Consider the following $AR(2)$ model

$$x_t = 0.045 + 1.182x_{t-1} - 0.219x_{t-2} + a_t$$

where a_t is normally distributed with mean 0 and standard deviation 0.448. Simulate this processes 100 times and repeat the analysis. Report your finding.