Financial Time Series

Examples in Chapter 3

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OUTLINE

1. Unit Root Test:
   - The UK interest rate spread: Example 3.1
   - The dollar/sterling exchange rate: Example 3.1
   - The dividend yield on the UK All share index: Example 3.1
2. DS versus TS:
   The UK FTA All Share index. Example 3.2.
3. More than one unit root:
   UK interest rate. Example 3.3.
4. Structural Breaks:
   US stock prices. Example 3.4.
5. Persistence and Mean Reversion:
   US stock prices. Example 3.6.
6. Long memory and Fractional Differencing:
   exchange rates and stock returns. Example 3.7.

Ex. 3.1: The UK interest rate spread

- Figure 3.6 shows plots of the UK short and long interest rates.
- Example 2.2: Try an $AR(2)$ process.
- Example 2.4: Try an $I(1)$ process without drift.
  Examination of SACF and SPACF of $\triangle x_t$ suggests that either $ARIMA(1,1,0)$ or $ARIMA(0,1,1)$ gives good fit.
- Example 2.7: Illustrate the difference between $AR(2)$ and $I(1)$ in terms of prediction.
  - $AR(2)$: The forecast error variances converge to the sample variance.
  - $I(1)$: The forecast error variances increase with h.
- Purpose: Apply a unit root test to discriminate between the two models.
- Fitted $I(1)$ model: $w_t = \Delta x_t$

$$\begin{aligned} w_t &= -0.0002(\pm.0198) + 0.201(\pm.043)w_{t-1} \\ &\quad +\hat{a}_t, \end{aligned}$$

$$\hat{\sigma} = 0.453$$

Or,

$$\begin{aligned} x_t &= -0.0002 + 1.201 x_{t-1} \\ &\quad -0.201 x_{t-2} + \hat{a}_t. \end{aligned}$$

- Fitted $AR(2)$ model:

$$\begin{aligned} x_t &= 0.045(\pm.023) + 1.182(\pm.043) x_{t-1} \\ &\quad -0.219(\pm.043) x_{t-2} + \hat{a}_t \end{aligned}$$

- Rewrite the above as

$$\begin{aligned} x_t &= 0.045(\pm.023) + 0.963(\pm.011) x_{t-1} \\ &\quad +0.219(\pm.043) \triangle x_{t-1} + \hat{a}_t. \end{aligned}$$

  – Test 1: $T = 526$,

  $$T(\hat{\phi} - 1) = T(0.963 - 1) = -19.5$$

  It is significant at the 2.5 per cent level.

  – Test 2:

  $$\tau_\mu = (0.963 - 1)/0.011 = -3.52$$

  It is significant at the 1 per cent level.

- Confirmation:
  Rewrite the above as

$$\begin{aligned} \triangle x_t &= 0.045(\pm.023) - 0.037(\pm.011) x_{t-1} \\ &\quad -0.219(\pm.043) \triangle x_{t-1} + a_t. \end{aligned}$$

– Non-parametric τ_μ statistic:

$$Z(\tau_\mu) = -3.49 \quad \text{with } \ell = 5$$

Reject a unit root at the 1 per cent level.

– Test of $\theta_0 = 0, \phi = 1$:

$$\Phi = 6.19$$

It is significant at the 2.5 per cent level.

- Conclude that the appropriate model for the spread is a stationary $AR(2)$ process.

Ex. 3.1: The dollar/sterling exchange rate

- Figure 2.13: Plots of daily observations of both the level and first differences of the dollar/sterling exchange rate from January 1974 to December 1994 (5192 observations).
- The differences are stationary.
  Suggest $I(1)$ process or a unit root.
- Test for a unit root.
- Fitted model:

$$\begin{aligned} \triangle x_t &= 0.0015(\pm.0009) - 0.00093(\pm.00052)x_{t-1} \\ &\quad +0.071(\pm.014) \triangle x_{t-1} + a_t. \end{aligned}$$

- $T = 5192$, $T(\hat{\phi} - 1) = -4.83$,
  $\tau_\mu = -1.79$, $\Phi = 1.93$,
  $Z(\tau_\mu) = -1.90$ with $\ell = 9$.
- All are clearly insignificant.
  Confirm that the appropriate model is indeed a random walk.

Ex. 3.1: The dividend yield (D/P) of the UK All share index

- Figure 3.4: Plots of D/P for the period January 1965 to December 1995.
- Example 2.6: Model it by an $ARIMA(1, 3)$ process.
- The figure does not show a trend but its wandering pattern could be a consequence of it being generated by an $I(1)$ process.
- Unit root versus $ARMA(1, 3)$
- Try an ADF test with the lag augmentation $k = [T^{0.25}]$.
- $T = 372$, $k = 4$, $\tau_\mu = -3.46$,
  Significant at the 1 per cent level.
- Nonparametric test:
  $Z(\tau_\mu) = -3.26$ with $\ell = 5$.
  Significant at the 2.5 per cent level.
- Reject the null of a unit root in favor of the alternative that the dividend yield is stationary.

Ex. 3.2: Are UK equity prices trend or difference stationary?

- Figure 14: Suggest a pronounced tendency to drift upwards.
- Example 2.6: Use $ARIMA(3,1,0)$ to model the logarithms of the UK FTA All Share index.
- D(ifference)S versue T(rend)S
- Test the null hypothesis that the series contains a unit root against the alternative that it is generated as stationary deviations about a linear trend.
- Four possible models:

$$\begin{aligned}
\triangle x_t &= 0.128(\pm 2.59) + 0.00026(\pm 2.56)t \\
&\quad -0.0287(\pm .052)x_{t-1} + \sum_{i=1}^{3} \hat{\delta}_i \triangle x_{t-i} + \hat{a}_t \\
\triangle x_t &= 0.0043(\pm .67) + 0.00001(\pm 2.56)t \\
&\quad + \sum_{i=1}^{3} \hat{\delta}_i \triangle x_{t-i} + \hat{a}_t \\
\triangle x_t &= 0.0121(\pm .61) - 0.00087(\pm .27)x_{t-1} \\
&\quad + \sum_{i=1}^{3} \hat{\delta}_i \triangle x_{t-i} + \hat{a}_t
\end{aligned}$$

$$\begin{aligned}\triangle x_t &= 0.0069(\pm 2.15) + \sum_{i=1}^{3} \hat{\delta}_i \triangle x_{t-i} \\ &\quad + \hat{a}_t.\end{aligned}$$

- Model (i):
  A τ_τ test cannot reject the DS null.
- Model (ii):
  β_1 is found to be insignificant under this null.
- Model (iii):
  A τ_μ test cannot reject the null.
- Model (iv):
  A unit root cannot be rejected.
- Confirm that equity prices do follow an $I(1)$ process
- The model tried in example 2.6 is indeed the appropriate one.

Ex. 3.3: Do UK interest rates contain two unit roots?

- Figure 3.6: Plots of the UK short and long interest rates
- RS_t: short rate; $R20_t$: long rate
- differencing twice?
  Consider
  $\triangle^2 RS_t = -0.007(\pm.023) - 0.714(\pm.042)\triangle RS_{t-1}$
  and
  $\triangle^2 R20_t = 0.005(\pm.014) - 0.702(\pm.042)\triangle R20_{t-1}$.
- $\tau_\mu = -17.05$ and -16.83
  Reject the hypothesis of two unit roots.
- Observe that

$$\begin{aligned}\triangle^2 RS_t &= -0.137(\pm.053) - 0.017(\pm.006) RS_{t-1} \\ &\quad -0.708(\pm.042) \triangle RS_{t-1}, \\ \triangle^2 R20_t &= 0.077(\pm.040) - 0.008(\pm.004) R20_{t-1} \\ &\quad -0.699(\pm.042) \triangle R20_{t-1}.\end{aligned}$$

- For the estimates of β_1, $\tau_\mu = -2.69$ and -1.90.
  Provide no evidence against the hypothesis that both series contain a single unit root.

Ex. 3.4: Unit roots and structural breaks in US stock prices

- Figure 3.7: the logarithms of the nominal annual (January average) return on S&P stock index for the period 1872 to 1997
- Purpose of Studying this Example:
  Unit Roots and Structural Breaks
- Unit root test: $\tau_\tau = -1.15$
  There is no evidence ($\tau_{\tau,0.10} = -3.15$) to reject the null hypothesis that they are DS (difference stationary).
- Consider the possibility of both a change in level and an increase trend rate of growth of the series in the wake of the Great Crash of 1929.
- Set the break point $T_B = 1929$. Then

$$\begin{aligned} x_t &= 0.034(\pm.089) + 0.731(\pm.058)x_{t-1} \\ &\quad +0.0066(\pm.0017)t - 0.235(\pm.065)DU_t \\ &\quad +0.012(\pm.003)DT_t^* + 0.184(\pm.181)DTB_t \\ &\quad +0.128(\pm.088) \triangle x_{t-1} \end{aligned}$$

- The t-ratio for testing $\phi = 1$: -4.65

Significant at the 5 per cent level. (T_B/T is around 0.5.)

- θ, β and γ are all significant.
- Conclude that x_t may be generated by a segmented trend process.
- Segmented trend process:

$$\begin{aligned}
x_t &= 1.335(\pm.206) + 0.0175(\pm.0053)t \\
&\quad +0.0430(\pm.0074)DT_t^* - 0.346(\pm.156)DU_t \\
&\quad +u_t \\
u_t &= 0.883(\pm.089)u_{t-1} - 0.194(\pm.090)u_{t-2} + e_t,
\end{aligned}$$

where $\hat{\sigma}_e = 0.1678$.

  - The crash provokes a decrease of almost 25 per cent in the trend level.
  - Prices grew at a trend rate of growth of 1.75 per cent per annum up to 1929 and 6.05 per cent thereafter.
  - No evidence of any nonstationary.
- Determine the break point T_B by the data. It is found to be 1931.
- The significance of the estimate of θ implies

that the trend function is not continuous at the break point.

- Try LSTR model:

$$\begin{aligned} x_t = {} & 1.389(\pm.054) + 0.0171(\pm.0012)t \\ & -2.714(\pm.352)S_t(0.738, 0.637) \\ & +0.0418(\pm.0034)tS_t(0.738, 0.637)DT_t^* + u_t. \end{aligned}$$

  - midpoint of the smooth transition: 1951
  - The transition takes about six years to complete. ($\hat{\gamma} = 0.738$)
  - Apply a unit root test to the residuals, we obtain -5.27.
    Significant at the 5 per cent level.
  - Fit the residuals by the $AR(2)$ process

    $$u_t = 0.883(\pm.089)u_{t-1} - 0.194(\pm.090)u_{t-2} + e_t,$$

    where $\hat{\sigma}_e = 0.1596$.
  - The standard error is smaller than that obtained from the segmented trend model.

Ex. 3.5: Estimating expected real rate of interest.

- Data: the broadest-based stock index in the UK:
  Financial Times Actuaries (FTA) All Share Index
- Question: Model its return
- Figure 2.14: monthly observations from January 1965 to December 1995 ($T = 371$)
- Figure 3.4: monthly dividend yield (D/P) from 1965 to 1995
- Based on the plot in Figure 2.14, this series exhibit a prominent upward, but not linear, trend, with pronounced and persistent fluctuations about it, which increase in variability as the level of the series increases.
- Use logarithmic transformation and the transformed observations are shown in Figure 2.14.
- Effect of taking logarithms: linearize the trend and stabilize the variance

Fit the ARMA model to the data

- $Q(12) = 26.4$, significance level: 0.009
- Table 2.3: Give the SACF and SPACF up to $k = 12$.
- Both r_k and $\hat{\phi}_{kk}$ at lags $k = 1$ and 2 are greater than two standard errors.
- What can be done now?
  Try the ARMA process.
- Based on the plot, we actually use a stationary process to model a non-stationary series. What is our best strategy without knowing the non-stationarity?
- Choose an appropriate model among the ARMA based on certain objective.
- Specification of possible models:
  $\bar{p} = \bar{q} = 3$ based on the SACF and SPACF.
- AIC selects $ARMA(2, 2)$:

$$\begin{aligned} x_t &= 1.57(\pm.10) - 1.054(\pm.059)x_{t-1} \\ &\quad -0.822(\pm.056)x_{t-2} + \hat{a}_t \\ &\quad +1.204(\pm.049)\hat{a}_{t-1} + 0.895(\pm.044)\hat{a}_{t-2}, \\ \hat{\sigma} &= 5.89 \end{aligned}$$

- BIC selects $MA(1)$:

$$
\begin{aligned}
x_t &= 0.55(\pm.04) + \hat{a}_t + 0.195(\pm.051)\hat{a}_{t-1}, \\
\hat{\sigma} &= 5.98
\end{aligned}
$$

- Table 2.4: Give AIC and BIC for all possible models.

Ex. 2.7: ARIMA forecasting of financial time series

UK interest spread (Example 2.2)

$AR(2)$ model:

$$\begin{aligned} x_t &= 0.045(\pm.023) + 1.182(\pm.043)x_{t-1} \\ &\quad -0.219(\pm.043)x_{t-2} + \hat{a}_t \\ \hat{\sigma} &= 0.448 \end{aligned}$$

- Last two observations: $x_{T-1} = 1.63$ and $x_T = 1.72$
- Forecasts:

$$\begin{aligned} f_{T,1} &= 1.182x_T - 0.219x_{T-1} = 1.676 \\ f_{T,2} &= 1.182f_{T,1} - 0.219x_T = 1.604 \\ f_{T,3} &= 1.182f_{T,2} - 0.219f_{T,1} = 1.529 \\ \cdots &= \cdots . \end{aligned}$$

  It can be shown that the forecasts eventually tend to 1.216, the sample mean of the spread.

- The ϕ-weight:

$$\begin{aligned} \phi_1 &= \psi_1 = 1.182 \\ \phi_2 &= \psi_1^2 + \psi_2 = 1.178 \end{aligned}$$

$$\begin{aligned} \phi_3 &= \psi_1^3 + 2\psi_1\psi_2 = 1.134 \\ \phi_4 &= \psi_1^4 + 3\psi_1^2\psi_2 + \psi_2^2 = 1.082 \end{aligned}$$

The forecast error variances are

$$\begin{aligned} V(e_{T,1}) &= 0.448^2 = 0.201 \\ V(e_{T,2}) &= 0.448^2(1 + 1.182^2) = 0.482 \\ V(e_{T,3}) &= 0.448^2(1 + 1.182^2 + 1.178^2) = 0.761 \\ V(e_{T,4}) &= 0.448^2(1 + 1.182^2 + 1.178^2 \\ &\quad +1.134^2) = 1.019 \end{aligned}$$

The forecast error variances converges to the sample variance of the spread, 3.53.

$ARIMA(0,1,1)$ model:

- $\hat{\theta} = 0.2$ and $\hat{\sigma} = 0.452$.
- Last observation: $x_T = 1.72$
- Final residual: $\hat{a}_T = 0.136$
- Forecasts:

$$\begin{aligned} f_{T,1} &= 1.72 - 0.2(0.136) = 1.693 \\ f_{T,h} &= f_{T,1} = 1.693. \end{aligned}$$

The forecasts will not converge to the sample mean.

- The forecast error variances are

$$\begin{aligned} V(e_{T,h}) &= 0.452^2[1 + 0.64(h-1)] \\ &= 0.204 + 0.131(h-1). \end{aligned}$$

The forecast error variances increase with h.