Financial Time Series

Examples in Chapter 3

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OUTLINE

1. Unit Root Test:
   – The UK interest rate spread: Example 3.1
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   – The dividend yield on the UK All share index: Example 3.1

2. DS versus TS:
   The UK FTA All Share index. Example 3.2.

3. More than one unit root:
   UK interest rate. Example 3.3.

4. Structural Breaks:
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5. Persistence and Mean Reversion:

6. Long memory and Fractional Differencing:
   exchange rates and stock returns. Example 3.7.
Ex. 3.1: The UK interest rate spread

- Figure 3.6 shows plots of the UK short and long interest rates.
- Example 2.2: Try an $AR(2)$ process.
- Example 2.4: Try an $I(1)$ process without drift.
  Examination of SACF and SPACF of $\Delta x_t$ suggests that either $ARIMA(1, 1, 0)$ or $ARIMA(0, 1, 1)$ gives good fit.
- Example 2.7: Illustrate the difference between $AR(2)$ and $I(1)$ in terms of prediction.
  
  - $AR(2)$: The forecast error variances converge to the sample variance.
  - $I(1)$: The forecast error variances increase with $h$.

- Purpose: Apply a unit root test to discriminate between the two models.

- Fitted $I(1)$ model: $w_t = \Delta x_t$
  
  \[
  w_t = -0.0002(\pm 0.0198) + 0.201(\pm 0.043)w_{t-1} + \hat{a}_t, \]

  \[+ \hat{a}_t, \]
\[ \hat{\sigma} = 0.453 \]

Or,
\[
x_t = -0.0002 + 1.201 x_{t-1} - 0.201 x_{t-2} + \hat{a}_t.
\]

- Fitted AR(2) model:
\[
x_t = 0.045(\pm 0.023) + 1.182(\pm 0.043)x_{t-1} - 0.219(\pm 0.043)x_{t-2} + \hat{a}_t
\]

- Rewrite the above as
\[
x_t = 0.045(\pm 0.023) + 0.963(\pm 0.011)x_{t-1} + 0.219(\pm 0.043) \Delta x_{t-1} + \hat{a}_t.
\]

- Test 1: \( T = 526 \),
\[
T(\hat{\phi} - 1) = T(0.963 - 1) = -19.5
\]
It is significant at the 2.5 per cent level.

- Test 2:
\[
\tau_\mu = (0.963 - 1)/0.011 = -3.52
\]
It is significant at the 1 per cent level.

- Confirmation:
Rewrite the above as
\[
\Delta x_t = 0.045(\pm 0.023) - 0.037(\pm 0.011)x_{t-1} - 0.219(\pm 0.043) \Delta x_{t-1} + a_t.
\]
- Non-parametric $\tau_{\mu}$ statistic:

$$Z(\tau_{\mu}) = -3.49 \quad \text{with } \ell = 5$$

Reject a unit root at the 1 per cent level.

- Test of $\theta_0 = 0, \phi = 1$:

$$\Phi = 6.19$$

It is significant at the 2.5 per cent level.

- Conclude that the appropriate model for the spread is a stationary $AR(2)$ process.
Ex. 3.1: The dollar/sterling exchange rate

- Figure 2.13: Plots of daily observations of both the level and first differences of the dollar/sterling exchange rate from January 1974 to December 1994 (5192 observations).

- The differences are stationary.
  Suggest $I(1)$ process or a unit root.

- Test for a unit root.

- Fitted model:

  $\triangle x_t = 0.0015(\pm.0009) - 0.00093(\pm.00052)x_{t-1}$

  $+ 0.071(\pm.014) \triangle x_{t-1} + a_t.$

- $T = 5192$, $T(\hat{\phi} - 1) = -4.83,$
  $\tau_\mu = -1.79$, $\Phi = 1.93,$
  $Z(\tau_\mu) = -1.90$ with $\ell = 9.$

- All are clearly insignificant.
  Confirm that the appropriate model is indeed a random walk.
Ex. 3.1: The dividend yield (D/P) of the UK All share index

- Figure 3.4: Plots of D/P for the period January 1965 to December 1995.

- Example 2.6: Model it by an $ARIMA(1, 3)$ process.

- The figure does not show a trend but its wandering pattern could be a consequence of it being generated by an $I(1)$ process.

- Unit root versus $ARMA(1, 3)$

- Try an ADF test with the lag augmentation $k = \lceil T^{0.25} \rceil$.

- $T = 372$, $k = 4$, $\tau_\mu = -3.46$, Significant at the 1 per cent level.

- Nonparametric test:
  
  $Z(\tau_\mu) = -3.26$ with $\ell = 5$.
  Significant at the 2.5 per cent level.

- Reject the null of a unit root in favor of the alternative that the dividend yield is stationary.
Ex. 3.2: Are UK equity prices trend or difference stationary?

- Figure 14: Suggest a pronounced tendency to drift upwards.
- Example 2.6: Use ARIMA(3, 1, 0) to model the logarithms of the UK FTA All Share index.
- D(ifference)S versus T(rend)S
- Test the null hypothesis that the series contains a unit root against the alternative that it is generated as stationary deviations about a linear trend.
- Four possible models:

\[
\Delta x_t = 0.128(\pm 2.59) + 0.00026(\pm 2.56)t \\
\quad -0.0287(\pm .052)x_{t-1} + \sum_{i=1}^{3} \hat{\delta}_i \Delta x_{t-i} + \hat{\alpha}_t
\]

\[
\Delta x_t = 0.0043(\pm .67) + 0.00001(\pm 2.56)t \\
\quad + \sum_{i=1}^{3} \hat{\delta}_i \Delta x_{t-i} + \hat{\alpha}_t
\]

\[
\Delta x_t = 0.0121(\pm .61) - 0.00087(\pm .27)x_{t-1} \\
\quad + \sum_{i=1}^{3} \hat{\delta}_i \Delta x_{t-i} + \hat{\alpha}_t
\]
\[ \Delta x_t = 0.0069(\pm 2.15) + \sum_{i=1}^{3} \hat{\delta}_i \Delta x_{t-i} + \hat{a}_t. \]

- **Model (i):**
  A \( \tau_r \) test cannot reject the DS null.

- **Model (ii):**
  \( \beta_1 \) is found to be insignificant under this null.

- **Model (iii):**
  A \( \tau_\mu \) test cannot reject the null.

- **Model (iv):**
  A unit root cannot be rejected.

- **Confirm that equity prices do follow an \( I(1) \) process**

- **The model tried in example 2.6 is indeed the appropriate one.**
Ex. 3.3: Do UK interest rates contain two unit roots?

• Figure 3.6: Plots of the UK short and long interest rates

• $R S_t$: short rate; $R 20_t$: long rate

• differencing twice?

Consider

\[ \Delta^2 R S_t = -0.007(\pm .023) - 0.714(\pm .042) \Delta R S_{t-1} \]

and

\[ \Delta^2 R 20_t = 0.005(\pm .014) - 0.702(\pm .042) \Delta R 20_{t-1}. \]

• $\tau_\mu = -17.05$ and $-16.83$

Reject the hypothesis of two unit roots.

• Observe that

\[ \Delta^2 R S_t = -0.137(\pm .053) - 0.017(\pm .006) R S_{t-1} - 0.708(\pm .042) \Delta R S_{t-1}, \]

\[ \Delta^2 R 20_t = 0.077(\pm .040) - 0.008(\pm .004) R 20_{t-1} - 0.699(\pm .042) \Delta R 20_{t-1}. \]

• For the estimates of $\beta_1$, $\tau_\mu = -2.69$ and $-1.90$.

Provide no evidence against the hypothesis that both series contain a single unit root.
Ex. 3.4: Unit roots and structural breaks in US stock prices

- Figure 3.7: the logarithms of the nominal annual (January average) return on S&P stock index for the period 1872 to 1997

- Purpose of Studying this Example:
  Unit Roots and Structural Breaks

- Unit root test: $\tau = -1.15$
  There is no evidence ($\tau_{0.10} = -3.15$) to reject the null hypothesis that they are DS (difference stationary).

- Consider the possibility of both a change in level and an increase trend rate of growth of the series in the wake of the Great Crash of 1929.

- Set the break point $T_B = 1929$. Then

\[
x_t = 0.034(\pm 0.089) + 0.731(\pm 0.058)x_{t-1} \\
+ 0.0066(\pm 0.0017)t - 0.235(\pm 0.065)DU_t \\
+ 0.012(\pm 0.003)DT^*_t + 0.184(\pm 0.181)DTB_t \\
+ 0.128(\pm 0.088) \triangle x_{t-1}
\]

- The t-ratio for testing $\phi = 1$: $-4.65$
Significant at the 5 per cent level. \( (T_B/T \text{ is around 0.5.}) \)

- \( \theta, \beta \) and \( \gamma \) are all significant.

- Conclude that \( x_t \) may be generated by a segmented trend process.

- Segmented trend process:
  \[
  x_t = 1.335(\pm.206) + 0.0175(\pm.0053)t
  + 0.0430(\pm.0074)DT_t^* - 0.346(\pm.156)DU_t
  + u_t
  
  u_t = 0.883(\pm.089)u_{t-1} - 0.194(\pm.090)u_{t-2} + e_t,
  \]
  where \( \hat{\sigma}_e = 0.1678. \)

  - The crash provokes a decrease of almost 25 per cent in the trend level.
  - Prices grew at a trend rate of growth of 1.75 per cent per annum up to 1929 and 6.05 per cent thereafter.
  - No evidence of any nonstationary.

- Determine the break point \( T_B \) by the data. It is found to be 1931.

- The significance of the estimate of \( \theta \) implies
that the trend function is not continuous at the break point.

- Try LSTR model:
  \[ x_t = 1.389(\pm .054) + 0.0171(\pm .0012)t \]
  \[ -2.714(\pm .352)S_t(0.738, 0.637) \]
  \[ +0.0418(\pm .0034)tS_t(0.738, 0.637)DT_t^* + u_t. \]
  - midpoint of the smooth transition: 1951
  - The transition takes about six years to complete. \((\hat{\gamma} = 0.738)\)
  - Apply a unit root test to the residuals, we obtain \(-5.27\). Significant at the 5 per cent level.
  - Fit the residuals by the \(AR(2)\) process
    \[ u_t = 0.883(\pm .089)u_{t-1} - 0.194(\pm .090)u_{t-2} + e_t, \]
    where \(\hat{\sigma}_e = 0.1596\).
  - The standard error is smaller than that obtained from the segmented trend model.
Ex. 3.5: Estimating expected real rate of interest.

- Data: the broadest-based stock index in the UK: **Financial Times Actuaries** (FTA) All Share Index
- Question: Model its return
- Figure 2.14: monthly observations from January 1965 to December 1995 ($T = 371$)
- Figure 3.4: monthly dividend yield (D/P) from 1965 to 1995
- Based on the plot in Figure 2.14, this series exhibit a prominent upward, but not linear, trend, with pronounced and persistent fluctuations about it, which increase in variability as the level of the series increases.
- Use logarithmic transformation and the transformed observations are shown in Figure 2.14.
- Effect of taking logarithms: linearize the trend and stabilize the variance

Fit the ARMA model to the data
• $Q(12) = 26.4$, significance level: 0.009

• Table 2.3: Give the SACF and SPACF up to $k = 12$.

• Both $r_k$ and $\hat{\phi}_{kk}$ at lags $k = 1$ and 2 are greater than two standard errors.

• What can be done now?
  Try the ARMA process.

• Based on the plot, we actually use a stationary process to model a non-stationary series. What is our best strategy without knowing the non-stationarity?

• Choose an appropriate model among the ARMA based on certain objective.

• Specification of possible models:
  $\bar{p} = \bar{q} = 3$ based on the SACF and SPACF.

• AIC selects $ARMA(2, 2)$:

\[
x_t = 1.57(\pm.10) - 1.054(\pm.059)x_{t-1} \\
-0.822(\pm.056)x_{t-2} + \hat{a}_t \\
+1.204(\pm.049)\hat{a}_{t-1} + 0.895(\pm.044)\hat{a}_{t-2},
\]

$\hat{\sigma} = 5.89$
• BIC selects $MA(1)$:

$$x_t = 0.55(\pm 0.04) + \hat{a}_t + 0.195(\pm 0.051)\hat{a}_{t-1},$$

$$\hat{\sigma} = 5.98$$

• Table 2.4: Give AIC and BIC for all possible models.
Ex. 2.7: ARIMA forecasting of financial time series

UK interest spread (Example 2.2)

$AR(2)$ model:

$$
x_t = 0.045(\pm 0.023) + 1.182(\pm 0.043)x_{t-1} \nonumber$$

$$-0.219(\pm 0.043)x_{t-2} + \hat{a}_t \nonumber$$

$$\hat{\sigma} = 0.448 \nonumber$$

- Last two observations: $x_{T-1} = 1.63$ and $x_T = 1.72$

- Forecasts:

$$f_{T,1} = 1.182 x_T - 0.219 x_{T-1} = 1.676 \nonumber$$

$$f_{T,2} = 1.182 f_{T,1} - 0.219 x_T = 1.604 \nonumber$$

$$f_{T,3} = 1.182 f_{T,2} - 0.219 f_{T,1} = 1.529 \nonumber$$

$$\cdots = \cdots \nonumber$$

It can be shown that the forecasts eventually tend to 1.216, the sample mean of the spread.

- The $\phi$-weight:

$$\phi_1 = \psi_1 = 1.182 \nonumber$$

$$\phi_2 = \psi_1^2 + \psi_2 = 1.178 \nonumber$$
\[ \phi_3 = \psi_1^3 + 2\psi_1\psi_2 = 1.134 \]
\[ \phi_4 = \psi_1^4 + 3\psi_1^2\psi_2 + \psi_2^2 = 1.082 \]

The forecast error variances are

\[ V(e_{T,1}) = 0.448^2 = 0.201 \]
\[ V(e_{T,2}) = 0.448^2 (1 + 1.182^2) = 0.482 \]
\[ V(e_{T,3}) = 0.448^2 (1 + 1.182^2 + 1.178^2) = 0.761 \]
\[ V(e_{T,4}) = 0.448^2 (1 + 1.182^2 + 1.178^2 + 1.134^2) = 1.019 \]

The forecast error variances converges to the sample variance of the spread, 3.53.

**ARIMA(0, 1, 1) model:**

- \( \hat{\theta} = 0.2 \) and \( \hat{\sigma} = 0.452 \).
- Last observation: \( x_T = 1.72 \)
- Final residual: \( \hat{a}_T = 0.136 \)
- Forecasts:

\[ f_{T,1} = 1.72 - 0.2(0.136) = 1.693 \]
\[ f_{T,n} = f_{T,1} = 1.693. \]

The forecasts will not converge to the sample mean.
The forecast error variances are

\[ V(e_{T,h}) = 0.452^2[1 + 0.64(h - 1)] \]
\[ = 0.204 + 0.131(h - 1). \]

The forecast error variances increase with \( h \).