Comments to the book by D. B. West Introduction to Graph Theory Second Edition, Print 2

Gerard J. Chang, Mathematics, National Taiwan University

October 14, 2002

1 Fundamental Concepts

1.1 What is a graph?

Page 4 line 9 (and some places before Page 35): The term n-vertex graph is defined only until Page 35. (But Page 588 says that it appears in Page 34.)

Page 5 line 11: Why is the map coloring problem infamous?

Page 5 Definition 1.1.15: I don't like the terms path and cycle defined in this way. It is more natural to me considering a path as a walk (see Page 20) in which all vertices are distinct; and a cycle a trial in which all vertices are distinct except the first vertex is the same as the last. If we want to consider a walk (or trail, path, cycle) as a graph we may consider it as a subgraph of the original graph using the vertices and the edges in the sequence. In this way, it is more natural to say a walk contains a path as in Lemma 1.2.5 in Page 21.

Page 6 line 9: Connectivity is defined again in Page 21. (It is not necessary to define it at the moment.) If path is defined as in Page 5, it is better to say "each pair of vertices in G belongs to a path contained in G".

Page 6 line 12: The graph is Example 1.1.9 is also connected.

Page 6 Definition 1.1.17: I don't see why we need to restrict the definition to loopless graphs. We may count a loop on a vertex twice as in Definition 1.3.1 in Page 34. In this case, just put 2 on the diagonal of $A(G)$ for each loop; and a single 2 in $M(G)$. Notice that there are already several places use degree for general graphs before Page 34, eg, Page 20 line 17 and Page 22 line 19 etc.

Page 7 Definition 1.1.20 and Page 8 Remark 1.1.23: I prefer to define isomorphism for general graphs with a remark saying that we may use the statement in 1.1.20 for simple graphs.

Page 8 Proof of Proposition 1.1.24: Say a sentence that we write the proof for simple graphs.

Page 10 line 4: Do you want three dots or just a period?

1.2 Paths, cycles, and trials

Page 19 line –5: It is worth to mention that the mathematical induction is also equivalent to the well ordering property.

Page 23 line 3: It is worth mentioning that deleting a vertex may decrease the number of components by one if the deleted vertex is an isolated vertex.

Page 24 Lemma 1.2.15: I don't see where is the place in the proof using the condition that the walk is of odd length. It seems to me that if you delete "odd" from the all places, the proof is still good. However, the new lemma is of course not valid. This means that we in fact don't use the oddness condition in the proof. We may change "If W has no repeated vertex (other than first = last)," to "Suppose W has no repeated vertex (other than first = last). Since $l \geq 3$, W has also no repeated edges, and".

Page 27 lines 7 to 9: We may define degree for general graphs in Page 6.

Page 28 line –1: We may like to mention (probably, right after the definition of degree) that the degree of a vertex in a simple graph is the same as the number of its neighbors.

Page 29 proof of Proposition 1.2.29: The proof is fine, but we may also prove it by just using definition. Similar to the proof of Theorem 1.2.14, for any two vertices x and y in $G - u$, if an x, y-path in G contains u, say is of the form $\ldots, v, e, u, e', w. \ldots$, then replace the v–w portion by the v–w portion of P.

1.3 Vertex degrees and counting

Page 35 Definition 1.3.2: The term n-vertex graph have been used for many times. It is better to define it earlier.

Page 42 line 7: In Taiwan, high school students are quit familiar with the arithmetic-geometric inequality: $(x + y)/2 \geq \sqrt{xy}$ for any two nonnegative real numbers; with equality holds iff $x = y$.

It may be useful to mention that using the discrete approach, it is also possible to prove that the only simple graph achieving the upper bound is $K_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}$.

Page 50 Exercise 1.3.33: The formula is $n(G) = 1 + \binom{d(x)+1}{2}$ rather than $n(G) = 1 + \binom{d(x)}{2}$. Should say that k is the common degree of all vertices.

1.4 Directed graphs

Page 55 Definition 1.4.6: Same as for graphs, I also like paths and cycles are defined in the other ways.

Page 56 line 2: by treating the edges as unordered pairs $\Longrightarrow$ by treating the endpoints of the edges as unordered pairs

Page 56 lines 11 and -4: Subdigraph rather than subgraph? See Page 57 line -8.

Page 58 lines –10 and –9: The notion $n(D)$ has been used in the proof of Theorem 1.4.16 already.

Page 59 lines –6 and –5: This is not true as the digraph just above shows that the digraph could has loops but the corresponding bipartite graph is simple.

Page 60 line 5: I don't see why "(or graph)" is necessary.

Page 62 line –2: clique $\Longrightarrow$ complete graph
Notice that You use clique as a set of pairwise adjacent vertices.